$\qquad$

| Question: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | Total |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Points: | 20 | 10 | 5 | 15 | 5 | 10 | 5 | 15 | 85 |
| Score: |  |  |  |  |  |  |  |  |  |

There are 8 problems in this exam. The pages are printed on both sides. Make sure that you have them all.

Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of pages, but indicate what is where if you expect someone to look at it. Books, calculators, extra papers, and discussions with friends are not permitted.

Leave all answers in exact form (that is, do not approximate $\pi$, square roots, and so on.)

You have 90 minutes to complete this exam.
$\qquad$

1. Let $f(x)=x^{2}+3 x$ with domain all real numbers. Let $A=(1, f(1))$ and $B=(2, f(2))$. There is also the point $C=(x, f(x))$ with $x$ close to 1 .
(a) 5 points Calculate the slope of the line through $A$ and $B$.
(b) 5 points Give an equation for the line through $A$ and $B$.
(c) 5 points Explain that the slope of the line through $A$ and $C$ is given by

$$
\text { slope }=\frac{x^{2}+3 x-4}{x-1}
$$

(d) 5 points Calculate the slope of the tangent line to the graph of $f$ at $A$.
$\qquad$ Id: $\qquad$
2. (a) 5 points Calculate the limit

$$
\lim _{x \rightarrow 2} \frac{3 x^{2}-15 x+18}{x-2}
$$

(b) 5 points Calculate the limit

$$
\lim _{x \rightarrow 2} \frac{3 x^{2}-15 x+19}{x-2}
$$

3. (a) 5 points Explain whether the function

$$
f(x)= \begin{cases}\frac{x^{2}-3 x}{x^{2}-9} & x \neq 3 \\ 21 & x=3\end{cases}
$$

is continuous at $x=3$ or not.
$\qquad$
4. Given the function

$$
f(x)=\left[\frac{1}{(x-1)}+\frac{1}{(x-3)}\right]+\cos (\pi x)
$$

with domain the numbers between 1 and $3,1<x<3$.
(a) 5 points Calculate $f(2)$.
(b) 10 points Is there a solution, a number $x$ between 1 and 3 , of $f(x)=0$ ?
5. (a) 5 points Calculate

$$
\lim _{x \rightarrow \infty} \frac{3 x^{2}+21}{7 x^{4}+31 x} .
$$

6. (a) 5 points Calculate

$$
\lim _{x \rightarrow 0^{+}} e^{-\frac{1}{x}}
$$

(b) 5 points Calculate

$$
\lim _{x \rightarrow 0^{-}} e^{-\frac{1}{x}}
$$

7. (a) 5 points Explain in words

$$
\lim _{x \rightarrow \infty} f(x)=L
$$

$\qquad$
8. 15 points Sketch the graph of an example of a function $f$ that satisfies all of the following conditions.

- $f(0)=0$
- $f(7)=11$
- $\lim _{x \rightarrow 7^{-}} f(x)=3$
- $\lim _{x \rightarrow 7^{+}} f(x)=-3$
- $\lim _{x \rightarrow \infty} f(x)=0$
- $\lim _{x \rightarrow 2^{+}} f(x)=\infty$
- $\lim _{x \rightarrow 2^{-}} f(x)=-\infty$
- $\lim _{x \rightarrow 1^{+}} f(x)=\infty$
- $\lim _{x \rightarrow 1^{-}} f(x)=5$
- $\lim _{x \rightarrow-\infty} f(x)=-\infty$
- $f(1)=3$
- $f(2)=3$

