

Limits

Evaluate the following limits:

$$1) \quad \lim_{x \rightarrow 5} 3x + 12 =$$

$$2) \quad \lim_{x \rightarrow 5} \frac{x^2 - 4x + 1}{x^2 + 3} =$$

$$3) \quad \lim_{x \rightarrow 5} \frac{x^2 - 4x - 5}{3x - 15} =$$

$$4) \quad \lim_{x \rightarrow 4} \frac{2x^2 - 3x - 20}{3x^2 - 11x - 4} =$$

$$5) \quad \lim_{x \rightarrow 3} \frac{x^3 - 27}{x^2 - 9} =$$

$$6) \quad \lim_{x \rightarrow 2^+} \frac{5}{x - 2} =$$

$$7) \quad \lim_{x \rightarrow 2^-} \frac{5}{x - 2} =$$

$$8) \quad \lim_{x \rightarrow 2} \frac{5}{x - 2} =$$

$$9) \quad \lim_{x \rightarrow \infty} \frac{4x^2 - 8x + 1}{9x^2 + 3x + 5}$$

$$10) \quad \lim_{x \rightarrow -\infty} \frac{4x^2 - 8x + 1}{9x^2 + 3x + 5}$$

$$11) \quad \lim_{x \rightarrow \infty} \frac{x^2 - 11x + 2}{8x^3 + 9x - 7}$$

$$12) \quad \lim_{x \rightarrow \infty} \frac{x^2 - 11x + 2}{8x^3 + 9x - 7}$$

$$13) \quad \lim_{x \rightarrow \infty} \frac{x^4 - 2x + 9}{6x^2 + 8x - 5}$$

$$14) \quad \lim_{x \rightarrow 0} \frac{3 \cos x}{5} =$$

$$15) \quad \lim_{x \rightarrow 0} \frac{3 \sin x}{5} =$$

$$16) \quad \lim_{x \rightarrow 0} \frac{3 \sin x}{5x} =$$

$$17) \quad \lim_{x \rightarrow 0} \frac{2 \tan x}{7 \sin x} =$$

$$18) \quad \lim_{x \rightarrow 0} \frac{2 \tan 3x}{7 \sin 9x} =$$

$$19) \quad \lim_{x \rightarrow 0} \frac{(x+3)^2 - 9}{x} =$$

$$20) \quad \lim_{x \rightarrow 0} \frac{(4+x)^3 - 64}{x} =$$

1) For what value of a is $f(x)$ continuous for all real numbers if

$$f(x) = \begin{cases} x^2 - 4x + 1; & x < 3 \\ x^2 + ax + 2; & x \geq 3 \end{cases} ?$$

2) For what value of b is $f(x)$ continuous for all real numbers if

$$f(x) = \begin{cases} bx^2 + 6x - 4; & x < 2 \\ 3x^2 + bx + 1; & x \geq 2 \end{cases} ?$$

3) For what value of a and b is $f(x)$ continuous for all real numbers if

$$f(x) = \begin{cases} ax^2 + 2bx - 4; & x < 1 \\ x^2 + 4ax + b; & 1 \leq x \leq 2 \\ 3bx^2 - 6ax + 12; & x > 2 \end{cases} ?$$

4) For what value of a and b is $f(x)$ continuous for all real numbers if

$$f(x) = \begin{cases} 3x^2 - 5ax + 2b; & x < 1 \\ -x^2 + 2ax + b; & 1 \leq x \leq 2 \\ x^2 - 3ax + 3b; & x > 2 \end{cases} ?$$

5) For what value of a and b is $f(x)$ continuous for all real numbers if

$$f(x) = \begin{cases} ax^3 + 3bx^2 + 5; & x < -1 \\ x^2 - 2ax + 6b; & -1 \leq x \leq 1 \\ x^3 + ax + 2b; & x > 1 \end{cases} ?$$

For each of the following, find (a) the slope of the secant line at the given points; and (b) the slope of the tangent line at x using the Definition of the Derivative:

1) $f(x) = x^2 + 10$ at $(2, 14)$ and $(4, 26)$

2) $f(x) = 3x^2 - 7$ at $(2, 5)$ and $(6, 41)$

3) $f(x) = x^2 + x + 9$ at $(1, 11)$ and $(3, 21)$

4) $f(x) = -x^2 + 6x$ at $(-2, -16)$ and $(4, 40)$

5) $f(x) = x^3 + 4x^2 - 8$ at $(1, -3)$ and $(2, 16)$

6) $f(x) = \frac{1}{x} + 7$ at $(1, 8)$ and $(-1, 6)$

7) $f(x) = \frac{1}{2x+5}$ at $\left(1, \frac{1}{7}\right)$ and $\left(4, \frac{1}{13}\right)$

8) $f(x) = \sqrt{x-5}$ at $(9, 2)$ and $(14, 3)$

9) $f(x) = \frac{1}{x} + \sqrt{x}$ at $(1, 2)$ and $\left(4, \frac{9}{4}\right)$

10) $f(x) = \frac{1}{x^2}$ at $(1, 1)$ and $\left(2, \frac{1}{4}\right)$