

## Some problems of the type appropriate for MAT125 Final Part I

1. Compute the following limits. Please distinguish between “ $\lim f(x) = \infty$ ”, “ $\lim f(x) = -\infty$ ” and “limit does not exist even allowing for infinite values”.

(a)  $\lim_{x \rightarrow -1} x^2 + x - 1$

(b)  $\lim_{x \rightarrow -3} \frac{x^2 + 2x - 3}{x + 3}$

(c)  $\lim_{t \rightarrow 0} \frac{\sqrt{2-t} - \sqrt{2}}{t}$

(d)  $\lim_{x \rightarrow \infty} \cos(1/x)$

(e)  $\lim_{x \rightarrow \infty} \frac{x^3 + 2x + 1}{x^3 - 2x + 1}$

(f)  $\lim_{x \rightarrow \pi/2} \frac{\sin x}{2x - \pi}$

(g)  $\lim_{x \rightarrow 0} \frac{\tan 3x}{2x}$

2. For what value of  $k$  is the function

$$f(x) = \begin{cases} 3kx^2 + 4x + 1 & x < 1 \\ 2x^2 - 5kx - 1 & x \geq 1 \end{cases}$$

continuous?

3. Compute the derivatives of the following functions

(a)  $f(x) = x^3 - 12x^2 + x + 2\pi$

(b)  $f(x) = (2x + 1)\sin(x)$

(c)  $g(s) = \sqrt{1 + \ln(2s)}$

(d)  $h(t) = \frac{1 + e^t}{1 - e^t}$

(e)  $f(x) = (2x + 2)^{10}$

(f)  $a(x) = \arctan(x^2)$

4. On what interval(s) is  $f(x) = xe^{-x^2}$  increasing?

5. For what value(s) of  $x$  does  $f(x) = x^3 + 3x^2 - 72x - 9$  have an inflection point?

6. Let  $f(x) = -2x^3 + 6x^2 - 3$ .
- (a) Compute  $f'$ ,  $f''$ .
  - (b) On which intervals is  $f(x)$  increasing/decreasing?
  - (c) On which intervals is  $f(x)$  concave up/down?
  - (d) Find all critical points of  $f(x)$ . Which of them are local maximums? local minimums? neither? Justify your answer.
7. Stony Brook is going to build a new parking lot in the shape of a rectangle. It will be fenced in on three sides using 4000 feet of fence. The fourth side backs up to the woods and doesn't need a fence. What are the dimensions of the parking lot which has the maximum area?
8. A sphere is expanding at a rate of 48 cubic inches per second. At what rate is the radius growing when the radius is  $1/2$  inch?
9. Use differentials to approximate  $\sqrt{9.02}$ .
10. Write the equation of the line tangent to the curve  $y = \cos(2x)$  at  $x = \pi/6$ .
11. Find the value of  $\frac{dy}{dx}$  when  $x = -2$  and  $y = 1$  if

$$\frac{4}{x^2} + y^4 = 2.$$