MAT125 Spring 2014 Midterm #1

Your name:____________________________

TA's name:____________________________

Problem #1: Evaluate the following limits:

a) $\lim\limits_{x\to 7}\dfrac{x^2-2x-35}{2x^2-11x-21} = \lim\limits_{x\to 7}\dfrac{(x-7)(x+5)}{(x-7)(2x+3)} = \lim\limits_{x\to 7}\dfrac{x+5}{2x+3} = \dfrac{12}{17}$

b) $\lim\limits_{x\to -5}\dfrac{\frac{1}{5}+\frac{1}{x}}{5+x} = \lim\limits_{x\to -5}\dfrac{\frac{x+5}{5x}}{\frac{5+x}{1}} = \lim\limits_{x\to -5}\dfrac{x+5}{5x}\cdot\dfrac{1}{5+x} = \lim\limits_{x\to -5}\dfrac{1}{5x} = \dfrac{-1}{25}$

c) $\lim\limits_{x\to 3}\dfrac{\sqrt{3x-5}-2}{x-3} = \lim\limits_{x\to 3}\dfrac{\sqrt{3x-5}-2}{x-3}\cdot\dfrac{\sqrt{3x-5}+2}{\sqrt{3x-5}+2} = \lim\limits_{x\to 3}\dfrac{3x-5-4}{(x-3)(\sqrt{3x-5}+2)}$

$= \lim\limits_{x\to 3}\dfrac{3x-9}{(x-3)(\sqrt{3x-5}+2)} = \lim\limits_{x\to 3}\dfrac{3(x-3)}{(x-3)(\sqrt{3x-5}+2)}$

$= \lim\limits_{x\to 3}\dfrac{3}{\sqrt{3x-5}+2} = \dfrac{3}{4}$

Problem #2: Evaluate the following limits:

a) $\lim\limits_{x\to\infty} \frac{\cos x}{x^2}$ $\lim\limits_{x\to\infty} \frac{-1}{x^2} \le \lim\limits_{x\to\infty} \frac{\cos x}{x^2} \le \lim\limits_{x\to\infty} \frac{1}{x^2}$

$$0 \le \lim_{x\to\infty} \frac{\cos x}{x^2} \le 0$$

$$\therefore \lim_{x\to\infty} \frac{\cos x}{x^2} = 0$$

b) $\lim\limits_{x\to\infty} \frac{4x^3-5x+1}{7x^4+3x^2-x}$ $\sim \lim\limits_{x\to\infty} \frac{4x^3}{7x^4} = \lim\limits_{x\to\infty} \frac{4}{7x} = 0$

c) $\lim\limits_{x\to\infty} \frac{e^{-x}+5}{e^{-2x}+4}$ $= \lim\limits_{x\to\infty} \frac{\frac{1}{e^x}+5}{\frac{1}{e^{2x}}+4} = \frac{5}{4}$

Problem #3: Use the definition of the derivative to find $f'(x)$.

$$f(x) = 3x^2 - 5x + 4$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{[3(x+h)^2 - 5(x+h) + 4] - [3x^2 - 5x + 4]}{h}$$

$$= \lim_{h \to 0} \frac{[3(x^2 + 2xh + h^2) - 5x - 5h + 4] - 3x^2 + 5x - 4}{h}$$

$$= \lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 - 5x - 5h + 4 - 3x^2 + 5x - 4}{h}$$

$$= \lim_{h \to 0} \frac{6xh + 3h^2 - 5h}{h}$$

$$= \lim_{h \to 0} \frac{(6x + 3h - 5)(h)}{h}$$

$$= \lim_{h \to 0} 6x + 3h - 5 = 6x - 5$$

Problem #4: Use the definition of the derivative to find $f'(x)$ at $x = 2$.

$$f(x) = \frac{12}{3x-1}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad \text{so} \quad f'(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

$$f'(2) = \lim_{h \to 0} \frac{\frac{12}{3(2+h)-1} - \frac{12}{5}}{h} = \lim_{h \to 0} \frac{\frac{12}{5+3h} - \frac{12}{5}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{12(5) - 12(5+3h)}{5(5+3h)}}{h} = \lim_{h \to 0} \frac{\frac{-36h}{5(5+3h)}}{h}$$

$$= \lim_{h \to 0} \frac{-36h}{25+15h} \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{-36}{25+15h} = \frac{-36}{25}$$