**Part 1:** Skip this part if you have passed part 1.
Otherwise you must pass it for part 2 to be graded.

SOLUTIONS

**Name:** ____________________

1 point 1. Find $f'(x)$ if $f(x) = 5\left(x^3 - \cos x + \sqrt{x}\right)^{10}$

USE CHAIN RULE

$50\left(x^3 - \cos x + \sqrt{x}\right)^9\left(3x^2 + \sin x + \frac{1}{2\sqrt{x}}\right)$

1. ____________________

1 point 2. Evaluate $\lim_{x\to 0} \frac{\cos(3x)}{3x+6}$ $= \frac{\cos(0)}{0+6} = \frac{1}{6}$

2. $1/6$

1 point 3. Evaluate $\lim_{h\to 0} \frac{3(x+h)^3 - 3x^3}{h}$.

DERIVATIVE OF $3x^3$

(OR MULTIPLY OUT, CANCEL, PLUG IN)

3. $9x^2$

1 point 4. Find $\frac{dy}{dx}$ if $y = \frac{5x^3 - e^4}{\pi + \ln x}$

QUOTIENT RULE!

4. $\frac{(15x^2)(\pi + \ln x) - (5x^3 - e^4)\left(\frac{1}{x}\right)}{(\pi + \ln x)^2}$

1 point 5. Write the equation of the line tangent to $y = \ln x + \sin\left(\frac{\pi x}{2}\right)$ at $x = 1$.

AT $x=1$, $y = \ln 1 + \sin(\pi/2) = 0 + 1 = 1$

$\frac{dy}{dx} = \frac{1}{x} + \frac{\pi}{2}\cos\left(\frac{\pi x}{2}\right)$,

AT $x=1$, THIS IS $1 + \frac{\pi}{2}\cdot 0 = 1$

LINE IS

5. $y = 1 + (x-1)$ ie $y = x$

1 point 6. Compute the derivative with respect to $t$: $5t^5 - \frac{2t^2}{3} + \frac{5}{t} + \sin\left(\frac{\pi}{5}\right)$

REMEMBER, $\sin\left(\frac{\pi}{5}\right)$ IS A CONSTANT.

6. $25t^4 + \frac{4}{3}t - \frac{5}{t^2}$

**Part 1:** Skip this part if you have passed part 1.
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Name: SOLUTIONS

1 point 7. Find the $x$-coordinate of the point of inflection of $g(x) = 4x^3 + 18x^2 - 9x + 5$.

$g'(x) = 12x^2 + 36x - 9$

$g''(x) = 24x + 36.$

$g''(x) = 0 \Leftrightarrow x = \frac{-36}{24} = \frac{-3}{2}$

7. $-\frac{3}{2}$

1 point 8. What is the largest interval on which $f(x)$ is increasing if $f(x) = -2x^3 - 12x^2 + 72x + 100$?

$f'(x) = -6x^2 - 24x + 72 = -6(x+6)(x-2)$

$f'(x) > 0$ FOR $-6 < x < 2$

8. $-6 < x < 2$

1 point 9. Find $P'(4)$ if $P(x) = \left(3\sqrt{x} - \cot\left(\frac{\pi x}{8}\right)\right)\left(\frac{x^2}{8} - 1\right)$.

$P'(x) = \left(\frac{3}{2\sqrt{x}} + \csc^2(\pi/8 x) \cdot \pi/8\right)\left(\frac{x^2}{8} - 1\right) + \left(3\sqrt{x} - \cot\left(\frac{\pi}{8}x\right)\right)\left(\frac{x}{4}\right)$

$P'(4) = \left(\frac{3}{4} + 1 \cdot \frac{\pi}{8}\right)\left(\frac{16}{8} - 1\right) + (6 - 0)(1) =$

9. $\frac{3}{4} + \frac{\pi}{8} + 6$

1 point 10. Compute $F'(t)$ if $F(t) = e^{\frac{10}{t}} + e^{10t}$.

CHAIN RULE

10. $\frac{10}{t} e^{10/t} + 10t e^{10t}$

1 point 11. Find the $x$-coordinate of the local minimum of $8x^3 - 6x^2 - 12x + 5$. If there is none, write "None".

$f'(x) = 24x^2 - 12x - 12 = 12(2x+1)(x-1)$

$f''(x) = 24x - 12$

$f''(-\frac{1}{2}) < 0$, MAX, $f''(1) > 0$, MIN

11. $x = 1$

1 point 12. Compute the derivative of $\frac{\cos(10x)}{7} - \arcsin(10x)$.

CHAIN RULE

$\frac{-\sin(10x)}{7} \cdot 10 - \frac{1}{\sqrt{1 + (10x)^2}} \cdot 10 =$

12. $\frac{-10\sin(10x)}{7} - \frac{10}{\sqrt{1-100x^2}}$

**Part 2:** These will be graded **only** if you have passed part 1. **Name:** ______________

13. Find an antiderivative (that is, a function whose derivative is the given function) for each of the following functions:

3 points (a) $f(x) = 2x^3 - 6x^2 + 8x + e^2$

$\frac{1}{2}x^4 - 2x^3 + 4x^2 + e^2x + C$

(a) ______________

3 points (b) $g(x) = e^{2x} + \sin(3x)$

$\frac{1}{2}e^{2x} - \frac{1}{3}\cos(3x) + C$

(b) ______________

3 points (c) $h(x) = 5\sqrt{x} - \dfrac{3}{x}$

$\frac{15}{2}x^{3/2} - 3\ln|x| + C$

(c) ______________

3 points (d) $k(x) = \dfrac{5}{1+x^2}$

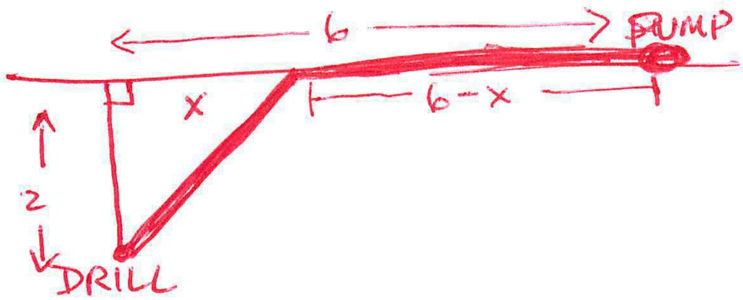
$5\arctan x + C$

(d) ______________

**Part 2:** These will be graded **only** if you have passed part 1. **Name:** ______________

| 12 points |
|---|

14. A company plans to build a pipeline from its drilling station, which is located in the ocean 2 miles south from a straight shoreline running east-west, to a pumping station which is located 6 miles east from the point on the shore directly opposite the drilling station. The pipeline will cost \$600/mile to run under the water and \$200/mile to run under the land. Where should the pipeline intersect the shore to be built for the minimum cost?




- LET $x$ BE THE DISTANCE FROM THE POINT ON SHORE DIRECTLY OPPOSITE THE DRILLING STATION TO WHERE THE PIPELINE HITS LAND.
- THE DISTANCE ON LAND IS $6-x$
- THE DISTANCE UNDERWATER IS $\sqrt{4+x^2}$

SO TOTAL COST OF PIPELINE IS

$$C(x) = 600\sqrt{4+x^2} + 200(6-x).$$

CRITICAL POINTS:

$$C'(x) = \frac{600x}{\sqrt{4+x^2}} - 200$$

$C'(x)=0$ IF $3x = \sqrt{4+x^2}$

OR $9x^2 = 4+x^2$

$8x^2 = 4$

$x = \pm\frac{1}{\sqrt{2}}$.

$x = \frac{1}{\sqrt{2}}$ IS RELEVANT CRIT. POINT.

MIN, SINCE $C'(x) < 0$ FOR $x < \frac{1}{\sqrt{2}}$

$C'(x) > 0$ FOR $x > \frac{1}{\sqrt{2}}$.

SHOULD HIT SHORE $\frac{1}{\sqrt{2}}$ MI FROM DRILLING STATION.

**Part 2:** These will be graded **only** if you have passed part 1. **Name:** ____________

8 points 15. (a) For the curve given by $x^3 - 3y^4 = 4x^2y^3 - 6$, find $dy/dx$ when $x = 1$ and $y = 1$.

BY IMPLICIT DIFF

$$3x^2 - 12y^3y' = 8xy^3 + 12x^2y^2y'$$

AT (1,1),

$$3 - 12y' = 8 + 12y'$$

$$-5 = 24y'$$

$$\frac{-5}{24} = y'.$$

5 points (b) Use your answer to the previous part to estimate the $y$-value of a point on the curve with $x = 1.2$.

TANGENT LINE AT (1,1) IS

$$y = 1 - \frac{5}{24}(x-1)$$

AT $X = 1.2 = \frac{6}{5}$, WE HAVE

$$y = 1 - \frac{5}{24}\left(\frac{1}{5}\right) = 1 - \frac{1}{24} = \frac{23}{24}$$

**Part 2:** These will be graded **only** if you have passed part 1. **Name:** ______________

16. Consider the function $f(x) = 4x^5 + 5x^4 - 40x^3$.

4 points (a) Find the $x$-values of all critical points of $f(x)$

$$f'(x) = 20x^4 + 20x^3 - 120x^2 = 20x^2(x^2 + x - 6)$$
$$= 20x^2(x+3)(x-2)$$

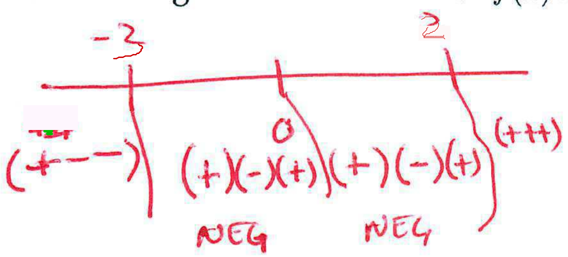
SO THERE ARE CRITICAL POINTS WHEN

$$20x^2(x+3)(x-2) = 0$$

THAT IS

$$x = 0,\ x = -3,\ \text{AND } x = 2.$$

4 points (b) State the largest interval on which $f(x)$ is decreasing.




IF $x < -3$, $f'(x) = (+)(-)(-) > 0$ SO INCREASING

IF $-3 < x < 0$, $f'(x) < 0$

IF $0 < x < 2$, $f'(x) < 0$

IF $2 < x$, $f'(x) > 0$.

$f$ IS DECREASING FOR $-3 < x < 2$

4 points (c) Give the $x$-values at which the absolute maximum and absolute minimum values of $f$ occur when $-1 \le x \le 3$.

JUST CHECK ENDPOINTS & CRITICAL POINTS.

$f(-1) = -4 + 5 + 40 = 41$ (IGNORE $-3$ SINCE NOT IN DOMAIN)

$f(0) = 0$

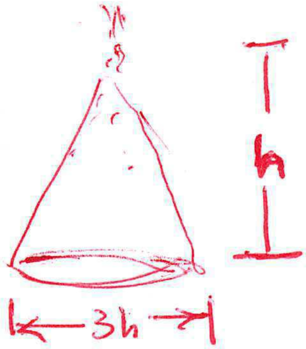
$f(3) = -112 < 0$

SO ABS MAX AT $x = -1$

ABS MIN AT $x = 3$.

**Part 2:** These will be graded **only** if you have passed part 1. **Name:** ______________

12 points 17. Sand is falling from a chute at the rate of 144ft$^3$ per minute, and is forming a conical pile whose diameter is always three times its height. Find the rate at which the height of the pile is growing when the pile is 1 foot high. You might find it useful to know that the volume of a cone of radius $r$ and height $h$ is $\pi r^2 h/3$, its surface area is $\pi r\left(r + \sqrt{r^2+h^2}\right)$, or that ice cream cone was invented in 1896 by Italo Marchiony. Or maybe not.




LET $h$ BE THE HEIGHT OF THE PILE, SO DIAMETER $= 3h$ (AND $r = 3h/2$)

SINCE THE SAND IS BEING ADDED AT 144 ft$^3$/MIN, $\frac{dV}{dt} = 144$.

WE WANT $dh/dt$ WHEN $h = 1$.

$$V = \pi r^2 h/3 = \pi\left(\frac{3h}{2}\right)^2\left(\frac{h}{3}\right) = \frac{3\pi}{4}h^3.$$

SO $$\frac{dV}{dt} = \frac{9\pi}{4}h^2\frac{dh}{dt}.$$

WHEN $h = 1$, WE HAVE

$$144 = \frac{9\pi}{4}\frac{dh}{dt}$$

SO $$\frac{2\cdot 288}{9\pi} = \frac{dh}{dt}$$

$$\frac{64}{\pi} = \frac{dh}{dt}$$

Name: ____________________

18. Let $R(x) = \dfrac{e^{2x} - 1}{\pi x}$.

$\boxed{\text{8 points}}$ (a) Find a value $k$ so that if we define $R(0) = k$, the resulting function is continuous. Fully justify your answer.

NOTE THAT $R(0) = \dfrac{1-1}{\pi \cdot 0} = \dfrac{0}{0}$, SO IT ISN'T A CONTINUOUS FUNCTION.

BUT USING L'HOPITALS (SINCE IT IS 0/0), WE GET $\displaystyle\lim_{x\to 0} \frac{e^{2x}-1}{\pi x} = \lim_{x \to 0} \frac{2e^{2x}}{\pi} = \frac{2}{\pi}$.

SO LET $R(0) = \dfrac{2}{\pi}$, AND

$$R(x) = \begin{cases} (e^{2x}-1)(\pi/x) & x \neq 0 \\ 2/\pi & x = 0 \end{cases}$$

IS CONTINUOUS.

$\boxed{\text{5 points}}$ (b) Is the function in the previous part differentiable at all values of $x$? Fully justify your answer.

YES

FOR $x \neq 0$, $R(x)$ IS RATIONAL AND THE DENOMINATOR IS NON ZERO, SO IT IS DIFFERENTIABLE.

TO GET $R'(0)$, WE LOOK AT

$$R'(0) = \lim_{h \to 0} \frac{R(0+h) - R(0)}{h} = \lim_{h\to 0} \frac{\frac{e^{2h}-1}{\pi h} - \frac{2}{\pi}}{h}$$

$$= \lim_{h \to 0} \frac{e^{2h} - 1 - 2h}{\pi h^2} \overset{\text{L'HOPITAL}}{=} \lim_{h \to 0} \frac{2e^{2h} - 2}{2\pi h} =$$



$\overset{\text{L'HOP}}{=} \displaystyle\lim_{h\to 0} \frac{4e^{2h}}{2\pi} = 2/\pi$

SO YES