MAT 125 Final Exam

Dec 14, 2009

Name: ______________________ **ID:** __________________ **Rec:** ____

R01	Th 9:50a	Tom	R10	W 10:40a	Yi	R20	M 2:20p	Jiansong	R30	M 5:20p	Panagiotis
R02	Tu 3:50p	Mark	R11	Th 8:20a	Carlos	R21	W 11:45a	Chris			
R03	W 10:40a	Ye Sle	R12	M 12:50p	Evan	R22	Th 2:20p	Jason	R32	W 3:50p	Vamsi
R04	M 11:45a	Jason	R13	F 12:50p	Vamsi	R23	Th 2:20p	Evan	R33	Tu 2:20p	Fred
R05	Tu 5:20p	Ye Sle	R14	W 11:45a	Ying	R24	M 11:45a	Fred	R34	F 11:45a	Jiansong
R08	M 10:40a	Nate	R15	M 2:20p	Chris	R25	Tu 9:50a	Carlos	R35	Th 3:50p	Marcelo
R09	M 6:50p	Yi	R16	Th 12:50p	Marcelo	R26	W 2:20p	Carlos			
			R17	Tu 11:20a	Mark	R27	W 10:40a	Ying	R91	MW 6:55p	Luca

There are 9 problems in this exam, printed on 8 pages (not including this cover sheet). Make sure that you have them all.

Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of pages, but indicate **clearly** what is where if you expect someone to look at it. If you actually read these instructions, write the words "why a duck?" at the bottom of this page and I'll give you two free points. Leave all answers in exact form (that is, do *not* approximate π, square roots, and so on.)

No calculators are permitted on this exam. **You may use a single sheet of notes**, provided that: (1) It is entirely handwritten and not a photocopy. (2) It is no larger than 8.5×11 inches, written on one side only. (3) It contains your name, written legibly. (4) You turn it in along with this exam. (5) Use of enchanted notes, (like Tom Riddle's diary in *Harry Potter and the Chamber of Secrets*) is permitted, but frowned upon because they tend to have unforseen consequences. **No other reference material is allowed.**

Question:	1	2	3	4	5	6	7	8	9	Total
Points:	16	12	15	10	10	16	10	11	10	110
Score:										

Do not open this booklet until told to do so

Name: ____________________ **Id:** ____________________

1. For each of the functions $f(x)$ given below, calculate the derivative $f'(x)$.

4 points (a) $f(x) = x^6 + \frac{x^2}{2} + \frac{1}{x}$

4 points (b) $f(x) = x^3 e^{-x^2}$

4 points (c) $f(x) = \arctan \sqrt{4x+1}$

4 points (d) $f(x) = \frac{(\sin 2x)^2 + (\cos 2x)^2}{e^{2x}}$

2. The derivative of a function $f(x)$ is

$$f'(x) = 3(x+3)(x-7)$$

$\boxed{\text{4 points}}$ (a) On what intervals is $f(x)$ increasing?

$\boxed{\text{4 points}}$ (b) On what intervals is $f(x)$ concave up?

$\boxed{\text{4 points}}$ (c) If $f(1) = 10$, what is $f(x)$?

3. Compute each of the limits below. If a limit does not exist, please distinguish between $+\infty$, $-\infty$, and "no limiting behavior (DNE)". Give some justification or show some work for each of your answers.

3 points (a) $\displaystyle\lim_{x\to 1^+} \frac{\cos\left(\frac{\pi}{2}x\right)}{\ln x}$

3 points (b) $\displaystyle\lim_{x\to +\infty} \cos\left(\frac{1}{x}\right)$

3 points (c) $\displaystyle\lim_{x\to \infty} \left[\, \ln(3+x) - \ln(x-3) \,\right]$

3 points (d) $\displaystyle\lim_{x\to 0} (1+3x)^{1/x}$

3 points (e) $\displaystyle\lim_{x\to +\infty} \frac{x^4+2x+1}{3x^4-2x+1}$

10 points

4. Let

$$P(x) = 1 + 9\left(A\frac{x - x^2}{1 + x}\right)^{50} \qquad \text{where } A = (1 + \sqrt{2})^2.$$

For x between 0 and 1, the function $P(x)$ has an absolute maximum value of 10. Your goal is to accurately determine the value of x where this maximum occurs. **Write a number x in the box at right**, with $0 < x < 1$.

$x =$ $\boxed{}$

Your score on this problem will be equal to $P(x)$ evaluated at that number, and *rounded down* to an integer. Be very careful in your work: being off by even one tenth can change your score from 10 to 1. Note also that the only "partial credit" assigned will be determined by the number in the box, not by your work.

10 points

5. Use Newton's Method to find a solution of the equation

$$x^3 + x = 1$$

starting with an initial guess $x_1 = 0$. Write the next two approximations given by Newton's method (that is, x_2 and x_3).

If any of your x_i include fractions with a denominator bigger than 8, you made an error.

For ease of grading, write your answers here, with your work below:

$x_2 =$ $\qquad\qquad\qquad\qquad$ $x_3 =$

6. The function $f(x)$ is defined as

$$f(x) = \begin{cases} x^2 \sin\left(\dfrac{1}{x}\right) & \text{when } \ x \neq 0 \\ 0 & \text{when } \ x = 0 \end{cases}$$

4 points (a) Show that f is continuous at $x = 0$.

4 points (b) Compute $f'(0)$. (Hint: use the definition of the derivative)

4 points (c) Compute $f'(x)$ when $x \neq 0$.

4 points (d) Is $f'(x)$ a continuous function? Fully explain your answer.

6 points 7. (a) Find all the critical numbers for the function

$$f(x) = \sin(2x) + \cos(2x) \qquad -\frac{\pi}{2} \le x \le \frac{\pi}{2}$$

and determine whether each is a relative minimum, relative maximum, or neither. You must justify your classification for full credit.

4 points (b) Find the absolute maximum and minimum values of $f(x)$ on the given interval.

8. Consider the curve given by $y^2 = x^3 + 3x^2$.

5 points

(a) Write the equation of the line tangent to this curve at the point $(1, 2)$.

3 points

(b) Using the result from the previous part, estimate the value of the y-coordinate when $x = 1.1$.

3 points

(c) For what (x, y) is the tangent line to the curve horizontal?

10 points 9. On a recent mission to the International Space Station, a water balloon was filled at a constant rate of $128 \frac{cm^3}{sec}$. Because of the lack of gravity, the balloon remained a perfect sphere the entire time. At what rate was the radius of the balloon increasing[1] when the radius was $4\,cm$?

[1]One or more of the following may be useful to you: The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$, and its surface area is $4\pi r^2$. The weight of $1\,cm^3$ of water is 1 gram. Heidemarie Stefanyshyn-Piper managed to dodge the balloon when it was thrown at her, but lost her toolbag. Water balloons can be dangerous in space— only trained professionals should use them.