MAT 125 First Midterm

February 22, 2016

Name: _____ **ID:** _____ **Rec:** ____

Question:	1	2	3	4	5	6	7	Total
Points:	20	20	20	15	10	10	16	111
Score:								

There are 7 problems in this exam. Make sure that you have them all.

Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of pages, but indicate what is where if you expect someone to look at it. **Books, calculators, extra papers, and discussions with friends are not permitted.** Telepathic communication with mathematically talented ducks is allowed, although you must also turn in the duck for grading along with your exam (geese and other fowl are not allowed). Please ensure the duck has a tag with your ID number on it.

Points will be taken off for writing mathematically false statements, even if the rest of the problem is correct.

Use non-erasable pen (not red) if you want to be able to contest the grading of any problems. Questions with erasures will not be regraded.

Leave all answers in exact form (that is, do *not* approximate π, square roots, and so on.)

You have 90 minutes to complete this exam.

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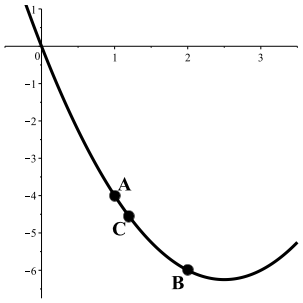
1. Let $f(x) = x^2 - 5x$.

   We will use the three points $A = (1, f(1))$, $B = (2, f(2))$, and $C = (x, f(x))$ with x close to 1.



5 points (a) Calculate the slope of the line passing through A and B.

5 points (b) Give an equation for the line through A and B.

5 points (c) Write an expression (depending on x) for the slope of the line through A and C.

5 points (d) Calculate the slope of the line tangent to the graph of f at the point A.

2. Compute each of the following limits. If the limit is not a finite number, please distinguish between $+\infty$, $-\infty$, and a limit which does not exist (DNE). Justify your answer, at least a little bit.

5 points (a) $\lim_{x\to 0} \dfrac{\sin x}{\tan x}$

5 points (b) $\lim_{x\to 1} \dfrac{x^2-1}{7x^2-7x}$

5 points (c) $\lim_{x\to 2} e^x \ln(x)$

5 points (d) $\lim_{x\to 1^+} \dfrac{x^2-9}{(x-3)(x-1)}$

3. More of the same: compute each of the following limits. If the limit is not a finite number, please distinguish between $+\infty$, $-\infty$, and a limit which does not exist (DNE). Justify your answer, at least a little bit.

5 points (a) $\lim_{x \to +\infty} \dfrac{4x^2 - 1}{8x^2 - 27x + 10}$

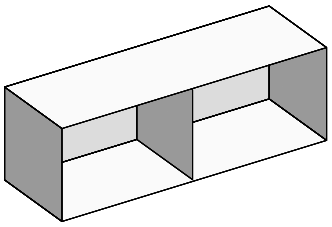
5 points (b) $\lim_{h \to 0} \dfrac{(x+h)^2 - x^2}{h}$

5 points (c) $\lim_{x \to 3} \dfrac{(x-3)^2}{|x-3|}$

5 points (d) $\lim_{x \to 1} \dfrac{\sqrt{x} - 1}{x - 1}$

15 points

4. An open, divided box is to be constructed from three square pieces of wood and three rectangular ones. The rectangular pieces will be used for the top, bottom and back, while the squares will form the ends and the divider.



Suppose the box is constructed from a total of 9 square feet of wood (and none is wasted). Let h represent the length of one of sides of the square pieces. Write an expression for the volume of the box in terms of h.

10 points 5. What value of k is necessary so that the function

$$f(x) = \begin{cases} k \tan\left(\frac{\pi}{4}x\right) & x < 1 \\ 3x^2 + x & x \geq 1 \end{cases}$$

is continuous for all positive values of x? Justify your answer fully.

10 points 6. Write a limit that represents the slope of the line tangent to the graph of the function

$$f(x) = \begin{cases} |x-2|^{\sin(\pi x)} & x \neq 2 \\ 1 & x = 2 \end{cases}$$

at $x = 2$. You **do not need to evaluate the limit.**

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16 points 7. Sketch the graph of a function $f(x)$ which satisfies all of the following properties:

- $f(2) = -1$
- $\lim_{x \to 2} f(x) \neq f(2)$
- $\lim_{x \to 1} f(x) = 1$
- $\lim_{x \to 0^+} f(x) = +\infty$
- $\lim_{x \to 0^-} f(x) = -\infty$
- $\lim_{x \to +\infty} f(x) = 0$
- $\lim_{x \to -\infty} f(x) = -\infty$
- $f'(-1) = 0$

Be sure to label all important values.