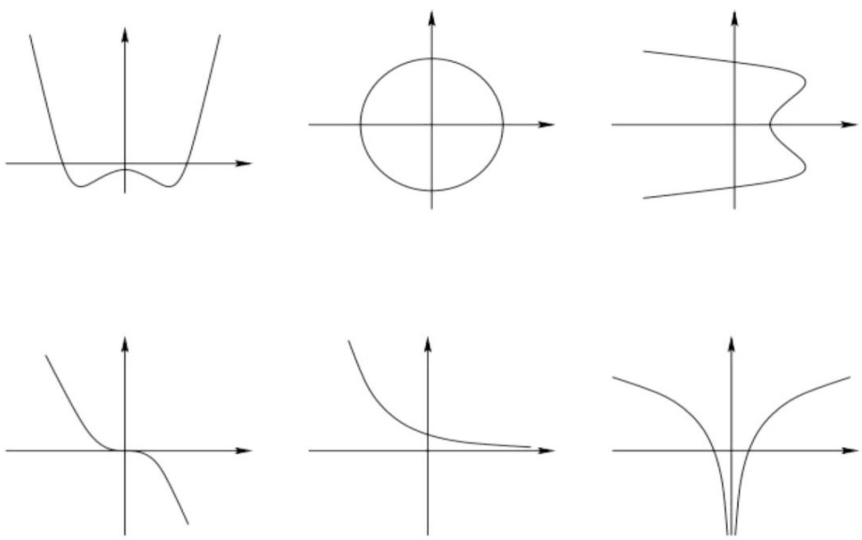
Diagnostic
Spring 2016

1. Specify whether the graph in each part represents a function and if it does, specify if the function is odd, even or neither:



2. For the parabola $y = -x^2 + 2x + 2$, find the coordinates of the vertex, an equation of the axis of symmetry, and the x and y intercepts. Draw the graph. Label your picture properly: indicate the vertex, the axis of symmetry, and the x and y intercepts.

3. Let $f(x) = 5^x, g(x) = 2x + 3$. Find $f \circ g, g \circ f$, and $g \circ g$.

4. Find the domain and range for each of the following functions. Also, write each as a composition of two functions or three functions where possible (do not choose the inner most function to $x$. We're not looking for triviality):

   (a) $f(x) = |x + 1|$

   (b) $y = 3^{x+1}$

   (c) $f(t) = \sin(\ln(t - 3))$

   (d) $g(u) = (u + 1)^{\frac{1}{4}}$

5. Simplify the following expressions:

   (a) $\log_3(\sqrt{27})$

(b) $2^{\log_{\frac{1}{2}}(\sqrt[3]{64})}$

6. In each of the following cases, find the domain of the given function, write it as a composition of two or three functions, and say whether the function is even, odd, or neither:
   (a) $x + \frac{1}{x}$
   (b) $\frac{x^3 - x}{x^3 + x}$
   (c) $|x|$
   (d) $\frac{x}{|x|}$
   (e) $\sqrt{x^4 + x^2 + 1}$

7. Simplify the following:
   (a) $27^{\frac{1}{3}}$
   (b) $1 + x^{\frac{1}{3}} + x^{\frac{2}{3}}$
   (c) $x^{\frac{1}{3}} x^{-\frac{1}{2}}$
   (d) $\frac{x^2 y^3}{(x^{-3} y^2)^{-3}}$
   (e) $(\frac{81x^5}{125y^3})^{\frac{1}{3}}$

8. Solve each of the following:
   (a) $\log_5(x - 1) = 2$
   (b) $\log_2(8x) = 5$
   (c) $\frac{\log_2(x)}{\log_2(3)} = 2$
   (d) $\log_2(x + 1) - \log_2(x - 1) = 2$

9. In each of the following cases, find the center of the given ellipse:
   (a) $4x^2 + 8x + y^2 - 2y = 11$
   (b) $x^2 + 2x + 4y^2 + 24y = -36$
   (c) $9x^2 + 36x + y^2 - 10y + 60 = 0$
   (d) $9x^2 - 54x + 4y^2 + 8y + 49 = 0$

10. In each of the following cases, find the following information:
    (a) Zeroes of $f$.
    (b) y-intercept.
    (c) Sign of the function between the zeroes.
    (d) The behavior of $f$ as $x \to \infty$.
    (e) Whether the function is odd, even, or neither.

(f) Give a rough sketch of the graph illustraing all of these features.

i. $f(t) = t(t^2 - 1)$

ii. $g(x) = x^3 - 9x$

iii. $h(u) = u^4 - 1$

iv. $j(x) = x^4 - 5x^2 + 4$

v. $k(n) = n^4 - 5n^3 + 4n^2$

11. Justify/Prove the following:

(a) $\cos^2(\theta) + \sin^2(\theta) = 1$.

(b) $\cos(-\theta) = \cos(\theta)$

(c) $\sin(-\theta) = -\sin(\theta)$

(d) $\tan(-\theta) = \tan(\theta)$

12. Let $\theta$ be an angle such that $\frac{\pi}{2} < \theta < \pi$ and $\sin(\theta) = \frac{2}{5}$. Find $cos(\theta)$.

13. In each of the following cases, convert the given degree measure of an angle to the corresponding radian measure of an angle:

(a) $30°$

(b) $75°$

(c) $-120°$

(d) $200°$

(e) $(\frac{200}{\pi})°$

(f) $285°$

(g) $-780°$

(h) $135°$

14. In each case, the cosine of $2\theta$ is given and an interval of $\theta$ is given. Find a quadratic equation satisfied by $cos\theta$ and $sin\theta$, and then solve the equation.

(a) $\cos(2\theta) = \frac{\sqrt{2}}{2}$, $\theta \in [0, \frac{\pi}{2})$

(b) $\cos(2\theta) = \frac{3}{4}$, $\theta \in [-\frac{\pi}{2}, 0)$

(c) $\cos(2\theta) = \frac{1}{2}$, $\theta \in (0, \frac{\pi}{2}]$

(d) $\cos(2\theta) = \sqrt{2 - \frac{\sqrt{2}}{2}}$, $\theta \in [0, \frac{\pi}{2}]$

(e) $\cos(2\theta) = 1$, $\theta \in (\frac{\pi}{2}, \pi]$

15. Verify each statement:

(a) $\cos(3\theta) = 4\cos^3\theta - 3\cos\theta$

(b) $\sin(3\theta) = -4\sin^3\theta + 3\sin\theta$

(c) $\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$

(d) $\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

(e) $\sec(2\theta) = \frac{\sec^2\theta}{2-\sec^2\theta}$

(f) $\csc(2\theta) = \frac{1}{2}\sec\theta\csc\theta$

(g) $\frac{\sin(2\theta)}{1+\cos(2\theta)} = \frac{1-\cos(2\theta)}{\sin(2\theta)}$

(h) $\sin(2\theta) = \frac{2\tan\theta}{1+\tan^2\theta}$

(i) $\cos(2\theta) = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

16. Compute each of the following values (no calculator work):

(a) $\sin^{-1}(\frac{1}{2})$

(b) $\cos^{-1}(\frac{1}{2})$

(c) $\tan^{-1}(\sqrt{3})$

(d) $\sin^{-1}(0)$

(e) $\cos^{-1}(0)$

(f) $\tan^{-1}(1)$

(g) $\sin^{-1}(\frac{-1}{\sqrt{2}})$

(h) $\cos^{-1}(\frac{-1}{\sqrt{2}})$

(i) $\tan^{-1}(0)$

(j) $\sin^{-1}(\frac{-\sqrt{3}}{2})$

(k) $\cos^{-1}(\frac{-\sqrt{3}}{2})$

17. Reduce the following using methods such as polynomial long division, synthetic division, or another method you can think of:

(a) $\frac{2x^2+3x+1}{x}$

(b) $\frac{2x^2+3x+1}{x+1}$

(c) $\frac{x^2+x+1}{x^2-x+1}$

(d) $\frac{x^3+x^2+x+1}{x^2+x+1}$

(e) $\frac{x^2+2x+3}{3x-2}$

18. Let $f(x) = e^{\sin(3x)}$.

(a) Present $f$ as a composition of two functions. (Do not choose $x$ as one of your functions)

(b) Present $f$ as a composition of three functions. (Do not choose $x$ as one of your functions)

19. Solve the equation $\sin(e^x) = 0$.

20. Show that $f(x) = e^{\cos(x)}$ is a periodic function. Find its domain, range and sketch the graph.