Review for Final Exam MAT 125, Fall 2004

This is a review sheet for the final exam. Doing the problems given here should help you prepare for the final exam.

About 40% of the final will be material that was covered on midterms 1 and 2. Most of that material **is not covered** on this review sheet. You should make sure that you understand and can do all of the problems on the midterms. The following additional problems from the textbook may also be helpful:

p. 83 (T/F)	1, 3, 6, 8
p. 84	2, 14, 15, 21, 24
p. 175 (T/F)	2, 3, 7, 10, 15
p. 176	1, 7–15, 20, 27, 32, 34, 38, 46
p. 255 (T/F)	6, 8, 10, 11
p. 255-6	4–15, 27, 31, 36, 47, 49

Of course, you should also review/redo the assigned homework problems. The Review problems on pages 336-339 are also useful.

1. For each of the following functions, find the absolute maximum and minimum values for $f(x)$ in the given intervals. Also state the x value where they occur.

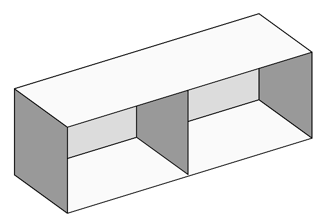
a. $\dfrac{\ln x}{x}$ for $1/3 \le x \le 3$.

b. $e^x - x$ for $-1 \le x \le 1$.

c. $x - \ln x$ for $1/2 \le x \le 2$.

d. $\sin(\cos(x))$ for $0 \le x \le 2\pi$.

2. An open, divided box is to be constructed from three square pieces of wood and three rectangular ones. The rectangular pieces will be used for the top, bottom and back, while the squares will form the ends and the divider. If the total area of the wood to be used is 9 sq. ft., what are the dimensions which will maximize the volume of the box?



3. For the functions $f(x) = xe^{-\frac{x^2}{2}}$ and $g(x) = e^{\frac{6x-9x^2}{4}}$ do the following:

a. Compute the first and second derivatives.

b. Find the intervals on which the function is increasing and decreasing.

c. Locate all local maximima and minima.

d. Locate all inflection points.

e. For which x is the function concave up?

f. Find all horizontal and vertical asymptotes, if any exist.

g. Use the above information to sketch the graph of the function.

4. A man starts running north at 10 km/h from a point P. At the same time, a car is 50 km. west of P and travels on a straight road directly east (towards P) at 60 km/h. How fast is the distance between the jogger and the driver changing 30 minutes later? Is the distance between them increasing or decreasing?

5. Find the following limits, if they exist. If the limit fails to exist, distinguish between $+\infty$, $-\infty$, and no limiting behavior (DNE).

- $\displaystyle\lim_{x\to 0} \frac{2\sin(5+x) - 2\sin(5)}{x}$
- $\displaystyle\lim_{x\to +\infty} \ln(2x^2+1) - \ln(x^2-3)$
- $\displaystyle\lim_{x\to 0} \ln[(e^x)^2] - \cos x$
- $\displaystyle\lim_{x\to 3+} x^2 \ln(x-3)$
- $\displaystyle\lim_{x\to +\infty} \frac{1}{2+\cos x}$
- $\displaystyle\lim_{x\to 0} x\sin(e^{1/x})$

6. A manufacturer wants to make a cylindrical can that has a volume of 400 cm^3. What should the radius of the base and the height be in order to minimize the amount of material needed (that is, in order to minimize the surface area)?

What if the top and bottom of the can are cut from rectangular pieces, and you cannot use the remaining corners for anything else?

7. Find antiderivatives for each of the following:

- $2x^7$
- $\sin(3x) + 3\cos(x)$
- $\dfrac{x^2+1}{x}$
- $\sqrt{2x^3}$
- e^{2x}
- $\dfrac{5}{x^2+1}$

8. A leaky oil tanker is anchored offshore. Because the water is very calm, the oil slick always stays circular as it expands, with a uniform depth of 1 meter. How rapidly is oil leaking from the tanker (in $\frac{m^3}{hr}$) if the radius of the slick is expanding at a rate of $8\frac{m}{hr}$ when the diameter is 20 meters?

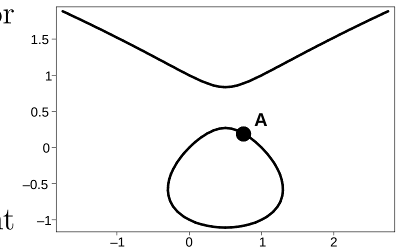
9. Use Newton's method on the function $g(x) = x^4 + 64x - 48$, starting with the initial approximation $x_1 = 2$, to find the third approximation x_3 to the solution of the equation $g(x) = 0$. Yes, I know $g(2)$ is a stupid initial guess (since $g(2) = 96$, not real close to 0). Give your answer to nine hundred billion decimal places. (Hint: if you need a calculator, you're probably doing it wrong, or you have trouble multiplying by 2.)

10. Joe Spacesuit is on a spacewalk floating freely outside the space station. He is at rest relative to the space station, and then turns on his jets, which give him an acceleration of 10 ft/sec^2. He accelerates at this constant rate for 2 seconds, and then shuts the jets off. If he started out 200 feet from the space station, how long does it take him to get there?

11. Consider the curve C which consists of the set of points for which

$$x^2 - x = y^3 - y$$

(see the graph at right).



a. Write the equation of the line tangent to C at the point $(1, 0)$.

b. Use your answer to part **a** to estimate the y-coordinate of the point with x-coordinate 3/4 marked A in the figure. Plug your estimate into the equation for C to determine how good it is.