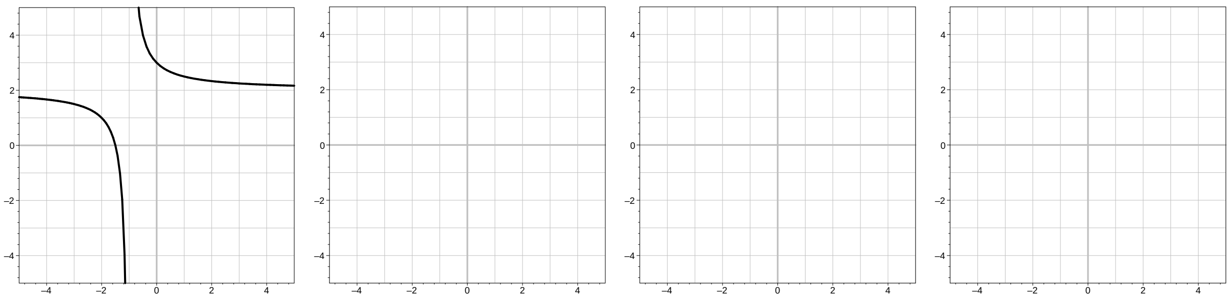
Sample First Exam MAT 125, Spring 1999

The midterm will be held on **Tuesday, February 23** at **8:30 pm** in **Javits**. Be sure to bring your Stony Brook ID card.

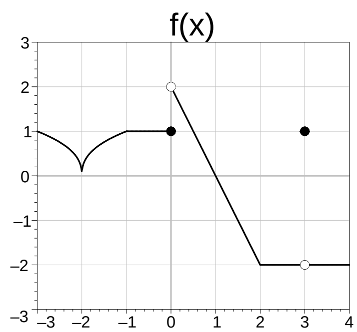
This sample represents the type of questions which will be on the midterm. The actual midterm will have a slightly different format, different questions, and may cover slightly different material. Just because something isn't asked here doesn't mean you aren't responsible for it. Remember that merely doing this sample is **not** sufficient preparation for the exam; you should also do most of the review problems at the end of chapters 1 and 2 in your text. Recall that the exam will cover through section 2.4 of the text.

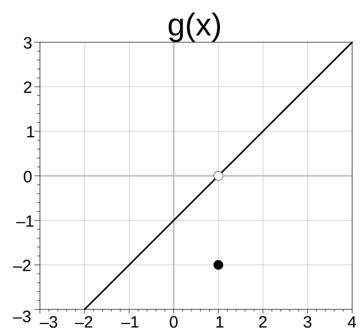
1. The graph of a function $f(x)$ is given at the left. Use it to draw the graphs of the functions $g(x) = f(-x)$, $h(x) = f^{-1}(x)$, and $k(x) = 2 - f(x)$.



2. Write $h(x) = 6x^4 - 2x^2 + 1$ as a composition of two functions f and g.

3. Use the graph of $f(x)$ and $g(x)$ below to compute each of the following quantities. If the quantity is not defininied, say so.



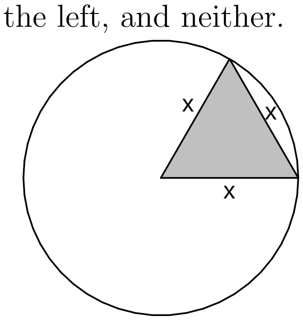


$f(0)$ $\qquad \lim\limits_{x\to 0} f(x)$ $\qquad \lim\limits_{x\to 0+} f(x)$ $\qquad \lim\limits_{x\to 0-} f(x)$

$\lim\limits_{x\to 1} g(x)$ $\qquad \lim\limits_{x\to -2} (f(x) + g(x/2))$ $\qquad \lim\limits_{x\to 1} \dfrac{f(x)}{g(x)}$ $\qquad \lim\limits_{x\to 3} (2f(x) - f(3))$

Identify all the points where $f(x)$ is continuous from the right, from the left, and neither.

4. An equilateral triangle is removed from a circle of radius x. One vertex of the triangle is at the center of the circle, and the other two are on its circumference (see the figure at right). Write a formula for the area of the part of the circle that remains; express this as a function of x, and state its domain.



5. Let $f(x) = \dfrac{1}{x+2}$ and $g(x) = \dfrac{x-1}{x}$. For each of the following, simplify as much as possible and state the domain.

$$f \circ g(x) \qquad g \circ f(x) \qquad f \circ f(x) \qquad g \circ g(x)$$

6. Compute each of the following limits, if they exist. If not, say so.

a. $\lim_{x \to 2} 3x^2 - x - 2$

b. $\lim_{x \to 2} \dfrac{x^2 + x - 6}{x - 2}$

c. $\lim_{q \to 2} \dfrac{2q^2 + 5}{q + 2}$

d. Suppose $x^2 + 1 \geq f(x) \geq 4x - 3$ for all x. Compute $\lim_{x \to 2} f(x)$.

e. $\lim_{y \to -3} |y + 3|$

f. $\lim_{h \to 0} \dfrac{(2+h)^2 - 2}{h}$

7. Sketch the graph of a function which satisfies all of the following conditions:

$$\lim_{x \to 1^+} f(x) = 2 \qquad \lim_{x \to 1^-} f(x) = 3 \qquad \lim_{x \to -2} f(x) = 1 \qquad f(1) = -1 \qquad f(-2) = 2$$

8. Simplify each of the following exactly:

$$\frac{8^{-1000}\sqrt{2^{10000}}}{16^{500}} \qquad \log_8 2 \qquad \log_6 2 + \log_6 3 \qquad \log_2 \frac{1}{16} \qquad \ln e^{\pi}$$

9. Write a formula for $f^{-1}(x)$ if $f(x) = \dfrac{x+5}{3x-4}$ and state the domain of f^{-1}.

10. The population (in thousands) of the Tzitzit bird is well described by a function of the form $P(t) = ae^{kt}$, where t is the time in years and a and k are constants. If the the population was 10 thousand when $t = 0$ and 300 thousand when $t = 3$, determine the constants a and k exactly. Then use the formula for $P(t)$ to find the population when $t = 4$.

11. Explain why the equation $x^5 - 3x + 1 = 0$ must have a solution with $0 < x < 1$.