

problem	1	2	3	4	5	6	7	Total
possible	16	12	16	20	10	6	20	100
score								

**Name:****Section:**

**Directions:** There are 7 problems on five pages in this exam. Make sure that you have them all. Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of pages, but indicate what is where if you expect someone to look at it. Books, extra papers, and discussions with friends are not permitted.

1. (16 points) Determine whether the following limits exist. If they do, find them. If the limit does not exist, distinguish between  $+\infty$ ,  $-\infty$ , and no limiting behavior (DNE). Justify your answers.

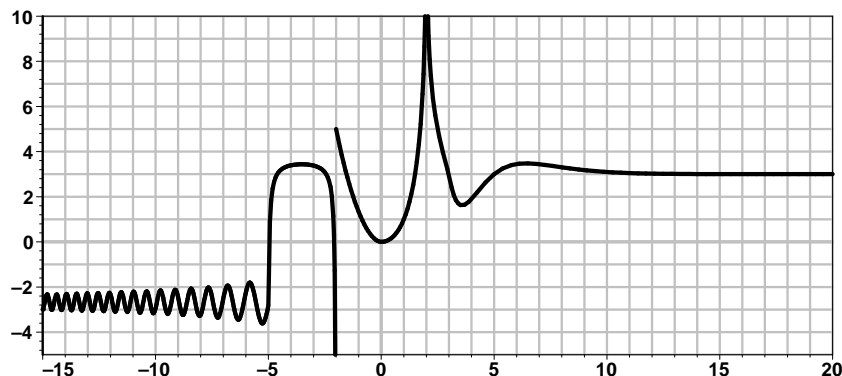
a.  $\lim_{x \rightarrow 1} \frac{2}{(x-1)^2}$

b.  $\lim_{x \rightarrow \infty} \frac{2x^3 - 7x + 1}{4x^3 + 8x^2 - 1}$

c.  $\lim_{x \rightarrow \infty} \cos(x)$

d.  $\lim_{x \rightarrow \infty} \sin(1/x)$

2. (12 points) The following is the graph of a function  $f(x)$ .



Using the graph, determine whether the following limits exist. If they do, calculate them. If the limit does not exist, distinguish between  $+\infty$ ,  $-\infty$ , and no limiting behavior (DNE).

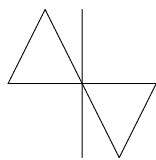
a.  $\lim_{x \rightarrow 2} f(x)$

b.  $\lim_{x \rightarrow -2} f(x)$

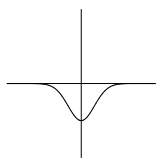
c.  $\lim_{x \rightarrow -2^-} f(x)$

d.  $\lim_{x \rightarrow \infty} \frac{f(x)}{x}$  (You may assume that for  $x$  large,  $|f(x)| < 4$ ).

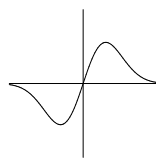
3. (16 points) Below are the graphs of four functions, labeled (a) through (d). Underneath each, write the number of the graph (1) through (6) which corresponds to its derivative.



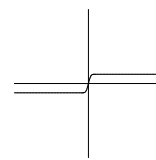
a



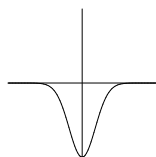
b



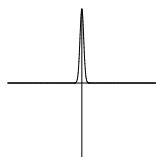
c



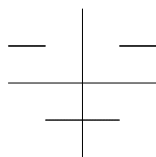
d



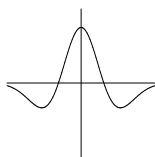
1



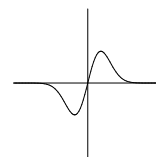
2



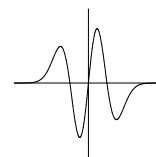
3



4



5



6

4. (20 points) For each function  $f(x)$  below, find its derivative  $f'(x)$ .

a.  $f(x) = x^2 e^x$

b.  $f(x) = 5e^x - 2x^2$

c.  $f(x) = \frac{x}{x^2 + 1}$

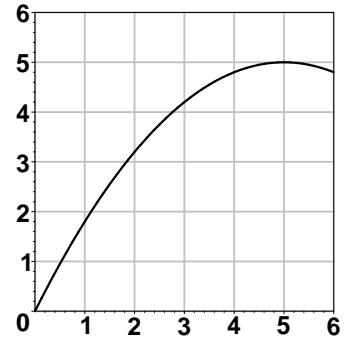
d.  $f(x) = e^{3\sqrt{x}}$

e.  $f(x) = \left(\frac{x-3}{x+2}\right)^2$

5. (10 points) At right is a graph of a function  $f(x)$ . Arrange the following quantities in increasing order:

$$1, f(0), f'(1), f'(3), f''(2).$$

Justify your answer.



6. (6 points) Let  $f(x) = x^x$ . Write a limit which represents  $f'(2)$ . (Note that you do **not** need to compute or simplify the limit).

7. (20 points) Let  $f(x) = e^{3x^2+1}$ .

Notice that  $f(x) = g(h(x))$ , where  $g(x) = e^x$  and  $h(x) = 3x^2 + 1$ .

a. Calculate  $f'(x)$ .

b. Calculate  $f''(x)$ .

c. For which values of  $x$  is  $f$  increasing? **Hint:** Remember that for any  $t$ ,  $e^t > 0$ .

d. Write the equation of the line tangent to  $f(x)$  at the point  $(1, e^4)$ .

e. Is  $f$  concave up at  $x = 1$ ? Justify your answer.