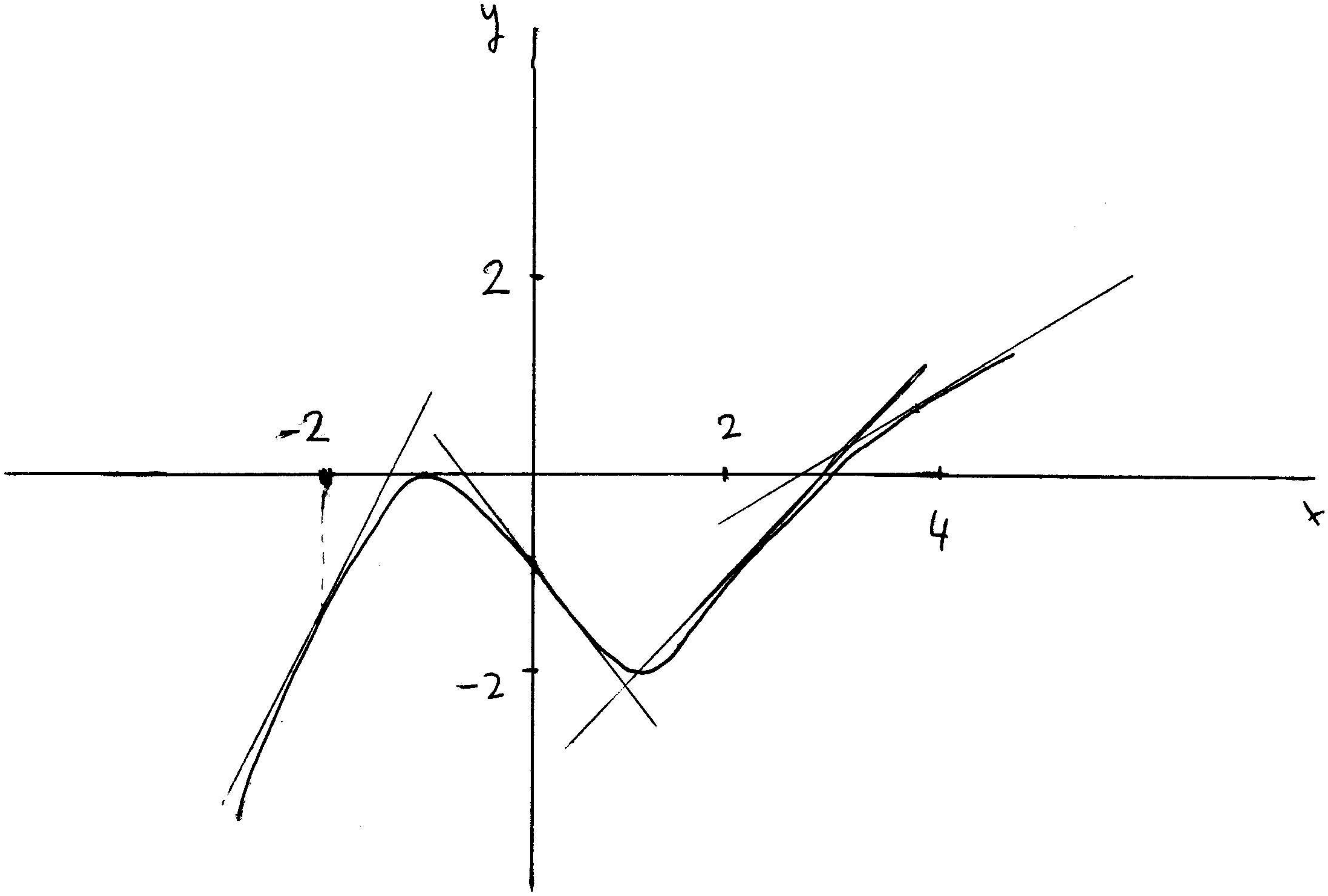
HW 6. Solutions

1. For the function g whose graph is given, arrange the following numbers in increasing order:

$$0, \ g'(-2), \ g'(0), \ g'(2), \ g'(4)$$



From the slopes of the corresponding tangents to the curve at points $x=-2$, $x=0$, $x=2$, $x=4$ we see

$$\boxed{g'(0) < 0 < g'(4) < g'(2) < g'(-2)}$$

2 a If $f(x) = 4x^2 - 4x$, find $f'(2)$.

$$f'(x) = 8x - 4 \Rightarrow \boxed{f'(2) = 8\cdot 2 - 4 = 12}$$

b Use $f'(2)$ to find an equation to the tangent line to the curve $y = 4x^2 - 4x$ at the point $(2,8)$.

The equation of the line is of the form

$$y = ax + b \quad \text{where} \quad a = f'(2) = 12.$$

To find b just substitute $x = 2$, $y = 8$:

$$8 = 12\cdot 2 + b = -16$$

$$\Rightarrow \boxed{y = 12x - 16}$$

3 If $f(t) = t^4 - 8t$ find $f'(a)$.

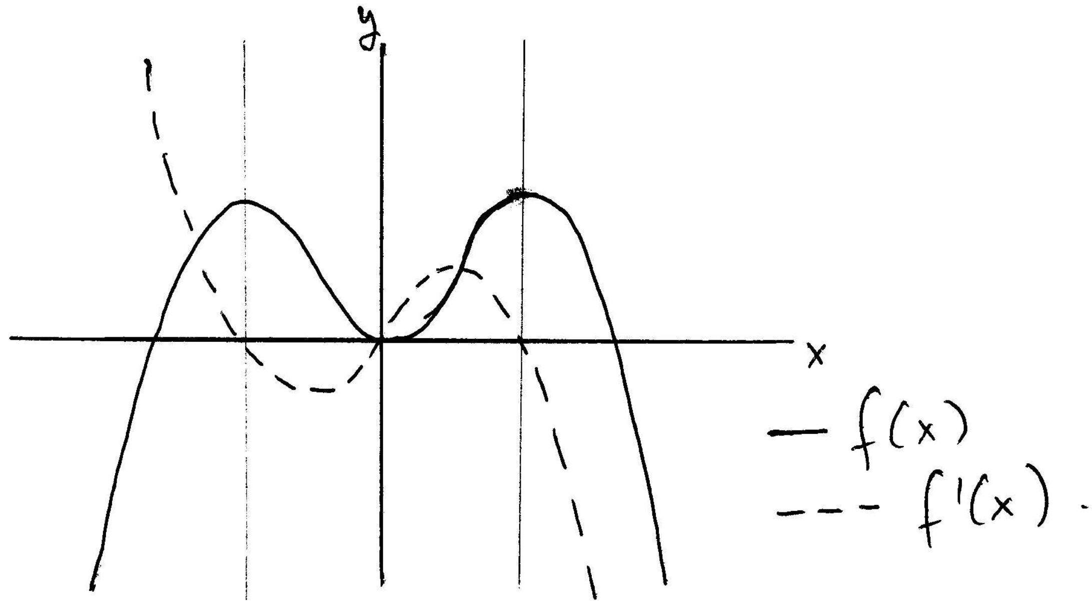
$$f'(t) = 4t^3 - 8 \Rightarrow \boxed{f'(a) = 4a^3 - 8}$$

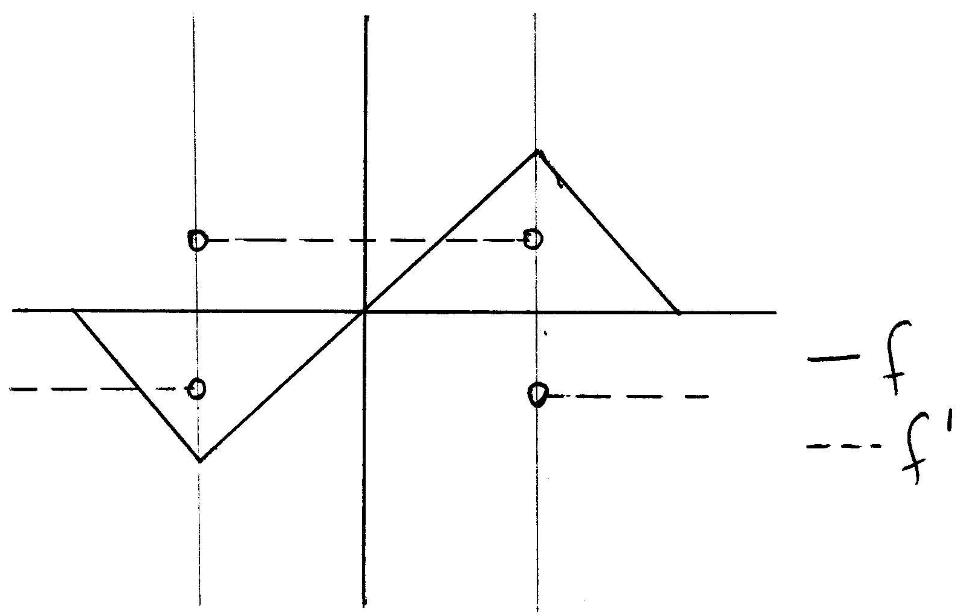
(4) Basic fact to know is that, in a given interval I,

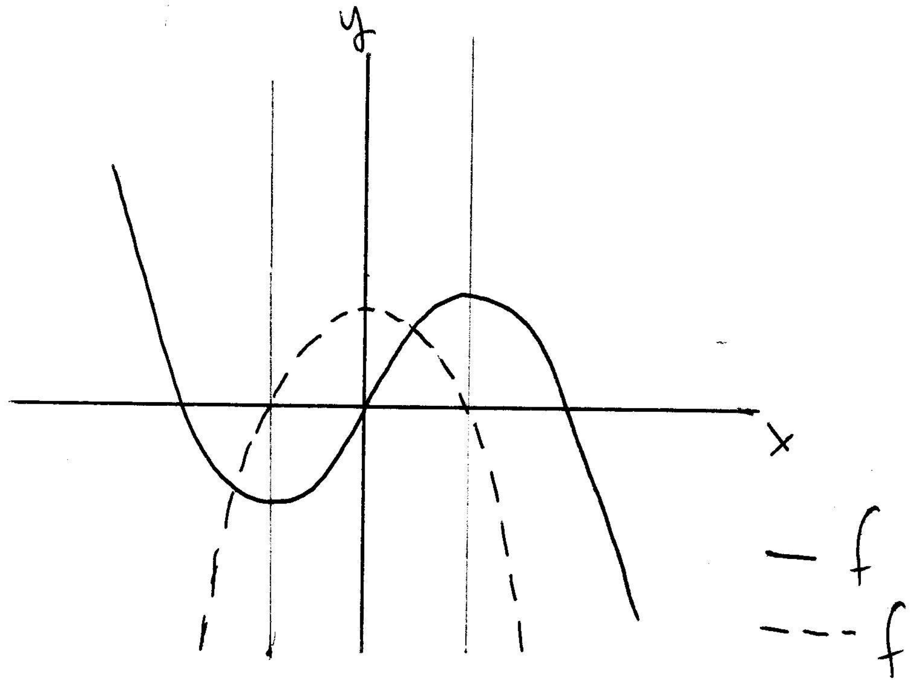
if f is increasing in $I \Rightarrow f'(x) > 0$, $x \in I$

f is decreasing in $I \Rightarrow f'(x) < 0$, $x \in I$

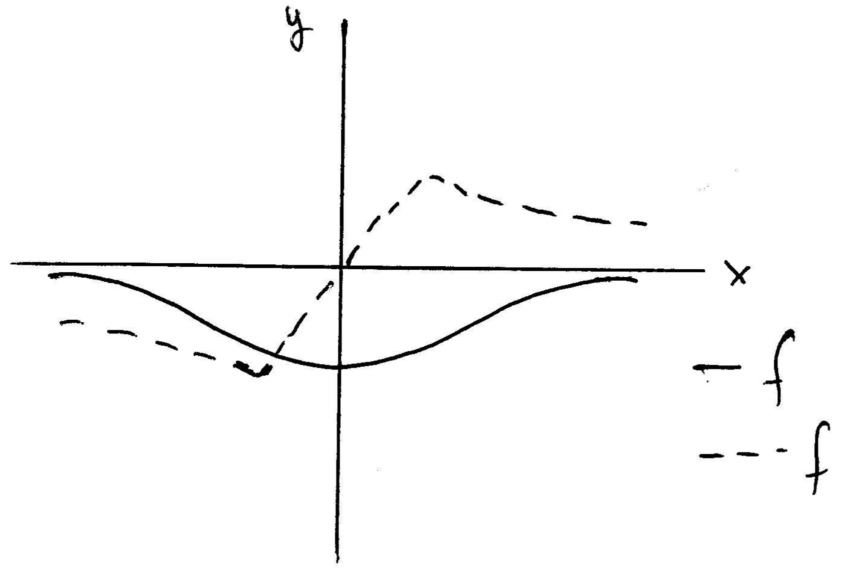
With this we have





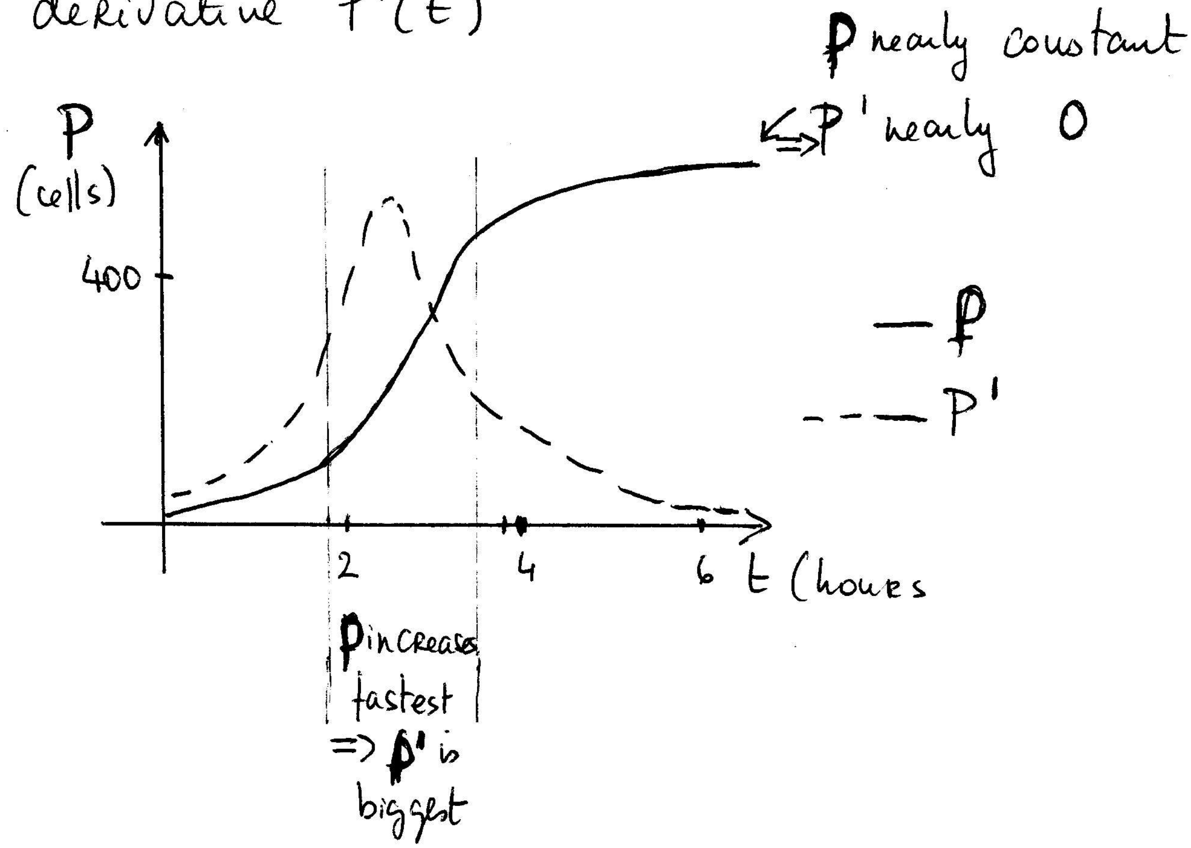


y
x
— f
--- f'



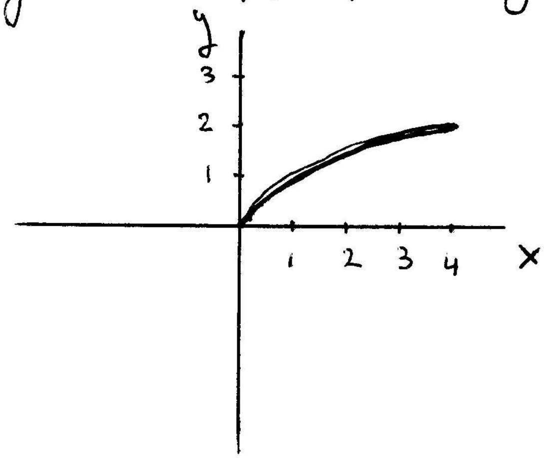
y
x
— f
--- f'

(5) Shown is the graph of the population function $P(t)$ for yeast cells in a lab culture. Graph the derivative $P'(t)$

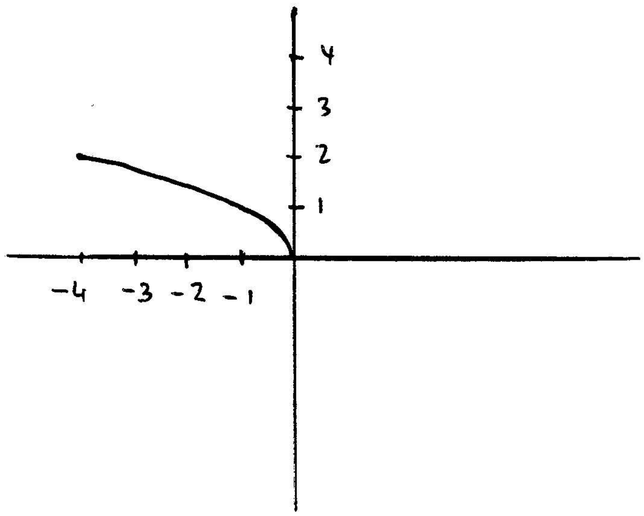


(6) Sketch the graph of $f(x) = \sqrt{8-x}$ by transforming the graph of $y = \sqrt{x}$ appropriately.

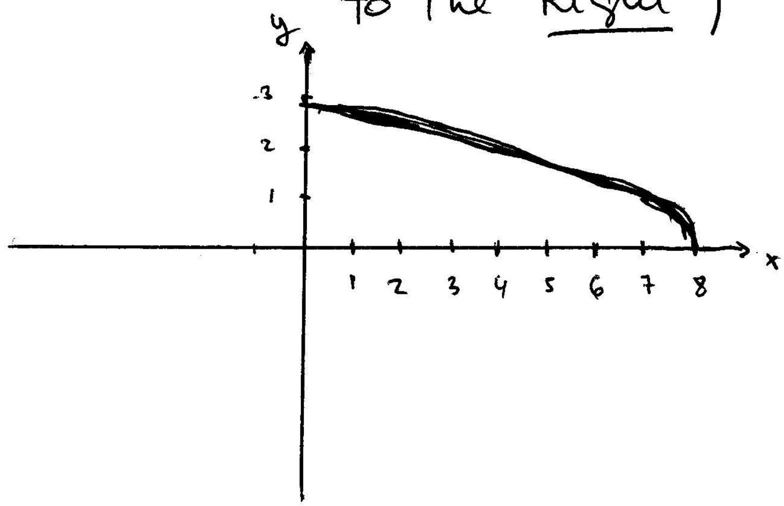
Graph of $y = \sqrt{x}$



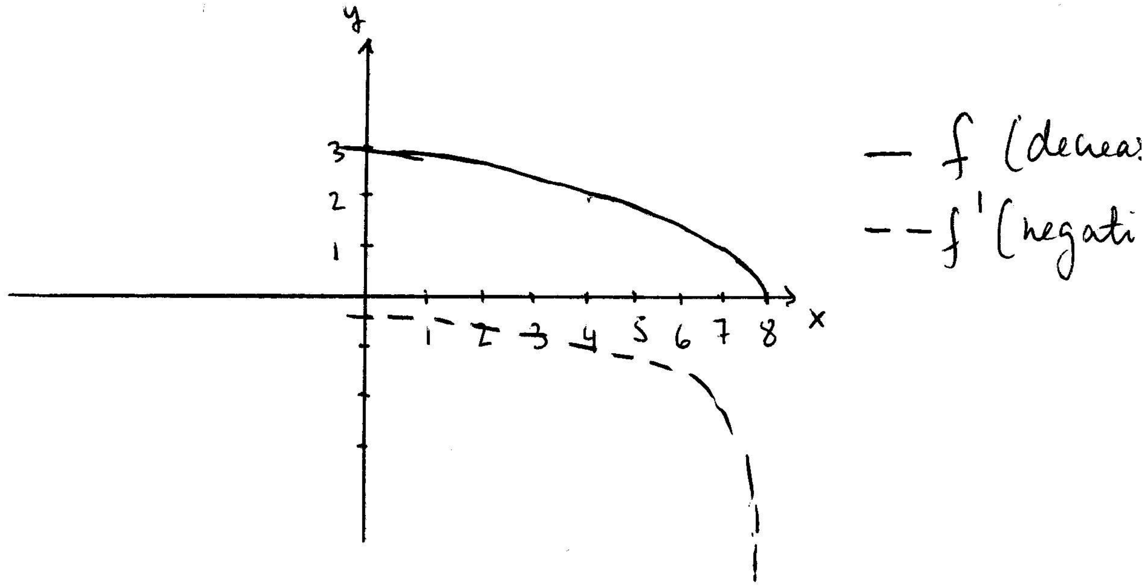
1st transformation : $x \longrightarrow -x \Rightarrow y = \sqrt{-x}$
(Reflect upon y-axis).



2nd transformation : $-x \longrightarrow 8-x \Rightarrow y = \sqrt{8-x}$
(move previous graph 8 units to the Right)



The graph of f^{-1} is



(c) Use the definition of a derivative to find $f'(x)$.

<u>Definition</u>: $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

in our case $f(x) = \sqrt{8-x}$

So $f(x+h) = \sqrt{8-(x+h)}$

$f(x) = \sqrt{8-x}$

Then $\dfrac{f(x+h) - f(x)}{h} = \dfrac{\sqrt{8-(x+h)} - \sqrt{8-x}}{h}$

So we have to compute the limit

$$\lim_{h\to 0} \frac{\sqrt{8-(x+h)} - \sqrt{8-x}}{h} = \lim_{h\to 0} \frac{\left(\sqrt{8-(x+h)} - \sqrt{8-x}\right)\left(\sqrt{8-(x+h)} + \sqrt{8-x}\right)}{h\left(\sqrt{8-(x+h)} + \sqrt{8-x}\right)}$$

$$= \lim_{h\to 0} \frac{[8-(x+h)] - [8-x]}{h\left(\sqrt{8-(x+h)} + \sqrt{8-x}\right)} = \lim_{h\to 0} \frac{-\cancel{h}}{\cancel{h}\left(\sqrt{8-(x+h)} + \sqrt{8-x}\right)}$$

$$= \lim_{h\to 0} \frac{-1}{\sqrt{8-(x+h)} + \sqrt{8-x}} = \frac{-1}{\sqrt{8-x} + \sqrt{8-x}} = \frac{-1}{2\sqrt{8-x}}$$

So

$$\boxed{f'(x) = \frac{-1}{2\sqrt{8-x}}}$$