MAT 123 First Midterm

September 30, 2015

Name: _____ **ID:** _____ **Rec:** ____

There are 13 problems in this exam, printed on 4 pages (not including this cover sheet). Make sure that you have them all.

The exam is in two parts: Part 1 consists of questions which should be *easy*. **You cannot get a grade of C or higher on this midterm unless you pass Part 1.**

Part I:

Question:	1	2	3	4	5	6	7	8	Total
Points:	2	2	2	2	2	2	2	2	16
Score:									

Part II:

Question:	9	10	11	12	13	Total
Points:	8	8	12	8	8	44
Score:						

Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of pages, but indicate what is where if you expect someone to look at it. **Books, calculators, extra papers, and discussions with friends are not permitted.** Answers and formulae that you have had permanently tatooed onto your skin may be referred to, although you must turn in a skin sample along with the exam to demonstrate that the tatoo is permanent.

Leave all answers in exact form (that is, do *not* approximate π, square roots, and so on) unless explicitly told otherwise. Algebraic simplification is typically not necessary, unless it is.

You have **90 minutes** to complete this exam. If you finish before that, it is advisable to use the remaining time to look back over your answers and make sure you still agree with them. If you *still* have more time, review all the choices you have made in your life up to this point, and make sure you still agree with them.

Part 1: minimum competence questions **Name:** _____

2 points 1. What is the value of $\cos\left(\frac{3\pi}{4}\right)$?

1. _____

2 points 2. What is the largest domain on which the function $f(x) = \frac{3-x}{\sqrt{1-x}}$ is defined?

2. _____

2 points 3. If $f(y) = y^2 - 3y$ and $g(x) = 2\sqrt{x}$, find $f(g(9))$.

3. _____

2 points 4. Suppose $f(x) = \begin{cases} x^2 - 4 & \text{if } x \leq 1 \\ 2x - 5 & \text{if } x > 1 \end{cases}$. Find all x such that $f(x) = 0$.

4. _____

2 points 5. Suppose that $\tan\alpha = \frac{10}{3}$ and $\pi < \alpha < \frac{3\pi}{2}$. Find $\sin\alpha$.

5. _____

2 points 6. Let $f(A) = A^2 + 5$. Find $f(x+h) - f(x)$; simplify your answer as much as possible.

6. _____

2 points 7. Give two angles x with $0 \leq x \leq \pi$ for which $\cos(2x) = \frac{\sqrt{3}}{2}$.

7. _____

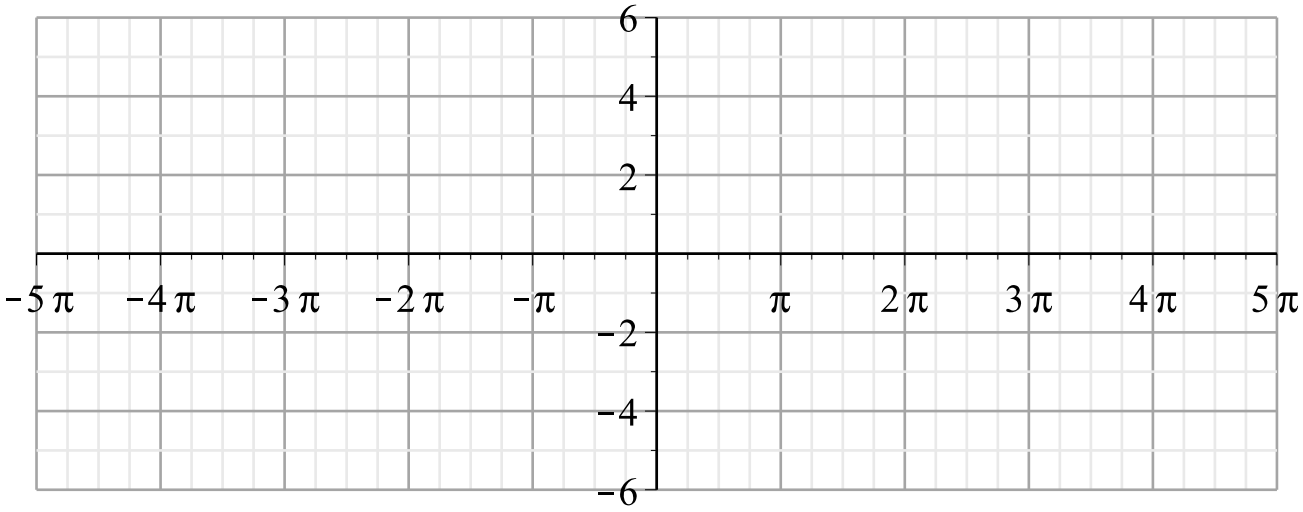
2 points 8. Suppose $f(x) = 3x^3 + 5$. Find $f^{-1}(x)$, if it exists. (If it does not exist, write DNE).

8. _____

Part 2: other questions

Name: _____

8 points 9. On the axes provided below, sketch the graph of $3\sin\left(x+\frac{\pi}{4}\right)-2$.



8 points 10. Find the smallest positive value of x so that $3\sin\left(x+\frac{\pi}{4}\right)-2=-\frac{1}{2}$.

Part 2: other questions

Name: _____

11. A swimming pool holds 50,000 gallons of water. Initially the pool is empty and it will filled using three different types of hoses. First, a hose that pumps pure water at a rate of 500 gallons per hour begins to fill the pool. When the volume of water in the pool is 10,000 gallons, the second hose is used in addition to the first hose. The second hose pumps slightly chlorinated water at 300 gallons per hour. When the volume of water in the pool is 30,000 gallons, the third hose is used in addition to the first two hoses. The third hose pumps water treated with an antibiotic at a rate of 200 gallons per hour.

8 points

(a) Write an expression for the function $V(t)$ which represents the volume of water at time t (in hours after the water was turned on).

4 points

(b) How long will it take (in hours) until the pool will be half-full (that is, until it contains 25,000 gallons of water)?

Name: _____

8 points 12. You are standing 50 meters from the base of a tall building, and you aim a laser pointer at the closest part of the top of the building. You measure that the laser pointer is 30° tilted from pointing straight up. You are holding the laser pointer 1 meter off the ground. How tall is the building?

8 points 13. Let $f(x) = \dfrac{3x-7}{2x+1}$. Find $f^{-1}(x)$.