

MAT 123 Midterm Exam

Name: _____

- Please read all instructions before beginning, and do not open the exam until you are told to do so.
- Put away all notes, books, calculators, etc. before beginning the exam, and place them under your desk. The only items on your desk should be this exam, pencils/pens, and an eraser.
- This exam has 6 questions, each with multiple parts. The point value for each question is shown below.
- The final page of the exam is left blank for scratch paper. You may tear this page from the exam booklet, but do not remove any other pages.
- You have 90 minutes to complete the exam.

Question	Points	Score
1	16	
2	7	
3	13	
4	5	
5	9	
6	15	
Total	65	

Question 1: (16 pts) Consider the function

$$f(x) = \frac{x^2 - 2x + 1}{3x^2 - 27}$$

(a) What is the domain of this function? Write your answer using interval notation.

(b) What are the zeros of this function?

(c) Describe the end behavior of this function.

(d) Find the y -intercept of the graph.

(e) Using the information from the previous parts of this problem, sketch a graph of this function. Your graph should include all asymptotes and the x and y intercepts. Some values of the function are computed below to help you graph the function -

$$f(-4) = \frac{25}{21} \quad f(-2) = \frac{-3}{5} \quad f(2) = \frac{-1}{15} \quad f(4) = \frac{3}{7}$$

Question 2: (7 pts) Consider the points $A = (-1, -4)$, $B = (7, -1)$ and $C = (1, -10)$ in the xy -plane.

- (a) Write an equation for the line that passes through the points B and C .
- (b) Write an equation for the line that is parallel to the line from part (a) and passes through the point A .
- (c) Write an equation for the line that is perpendicular to the line from part (a) and passes through the point A .

Question 3: (13 pts) Let $f(x) = 3^x$ and let $g(x) = 2x + 1$.

- (a) Write a formula for $h(x) = (g \circ f)(x)$. Using that formula, describe in words how the graph of $h(x)$ can be obtained from the graph of $f(x)$.

- (b) Draw the graph of $f(x) = 3^x$. Label at least two points on the graph. On the same set of axes, draw the graph of $h(x)$, and label at least two points on the graph.

(c) If you've drawn the graph of $h(x)$ correctly, you should notice that $h(x)$ is invertible. Draw the graph of $h^{-1}(x)$. Label at least two points on the graph, and make sure your graph has the proper domain and range.

(d) Find a formula for $h^{-1}(x)$. (Your formula should involve a logarithm.)

Question 4: (5 pts) Let $M(t) = -2t^2 - 8t - 13$.

(a) Write the equation for this quadratic in vertex form.

(b) Does this function have a maximum value? If so, what is it? Does this function have a minimum value? If so, what is it?

Question 5: (9 pts) Solve the following equations, or explain why there is no solution.

(a) $\log_3 \left(\frac{1}{9}\right) = \log_x 25$

(b) $2x^2 - 4x - 10 = 20$

(c) $3x^2 + 5x + 3 = 0$

Question 6: (15 pts) Consider the polynomial $p(x)$ defined by

$$p(x) = \frac{-1}{2}(x^2 - 4)(x^2 + 1)$$

- (a) Find the zeros of this polynomial.
- (b) Identify the degree and leading coefficient of $p(x)$.
- (c) Describe the end behavior of this polynomial.

(d) Find the values of $p(-1)$ and $p(1)$.

(e) Plot a graph of the polynomial. Your graph should have the proper x and y intercepts, end behavior, and number of “turns,” and should pass through the points you found in part (d).

SCRATCH PAPER