Practice Problems for Part Two of the MAT 123 Fall 2015 Final Exam

1) Use polynomial division to find the quotient:

a) $\dfrac{x^5+4x^4-15x^3+11x^2-3x-14}{x-2}$

b) $\dfrac{x^4+13x^3+25x^2-34x+40}{x+4}$

c) $\dfrac{x^5+2x^4-39x^3+64x^2-62x+16}{x+8}$

d) $\dfrac{x^4-2x^3-41x^2+-20x+50}{x+5}$

2) Solve for x:

a) $e^{2x}-4e^{x}-5=0$

b) $e^{2x}+4e^{x}-12=0$

c) $e^{x}+\dfrac{24}{e^{x}}=11$

d) $2e^{x}-\dfrac{2}{e^{x}}=-3$

e) $e^{2x}+8e^{x}+15=0$

3) Find all real values of *x* that solve the equation below. Please find the **exact** answer. That is, do not try to approximate logarithms, powers of *e*, square roots, etc.

$9^{x-4} = 10^{x}$

$4^{1-6x} = 8^{3x+2}$

$7^{6-4x} = e^{x}$

$5^{x+1} = 25^{3-3x}$

$2^{5+3x} = 3^{x}$

4a) If $f(x) = \frac{1}{4}x^2 - 4x - 2$ and $g(x) = \frac{16}{x}$, find:

(i) $f(g(8))$; (ii) $f(g(x))$; (iii) $g(f(x))$; (iv) $g^{-1}(x)$

b) If $f(x) = \frac{x^3 - 8}{7}$ and $g(x) = 3x - 2$, find:

(i) $g(f(4))$; (ii) $f(g(x))$; (iii) $g(f(x))$; (iv) $f^{-1}(x)$

c) If $f(x) = \sqrt{2x-4}$ and $g(x) = \sqrt[3]{2x+1}$, find:

(i) $f(g(13))$; (ii) $f(g(x))$; (iii) $g(f(x))$; (iv) $g^{-1}(x)$

d) If $f(x) = x^3 - 5$ and $g(x) = x^2 + 6x$, find:

(i) $f(g(-1))$; (ii) $f(g(x))$; (iii) $g(f(x))$; (iv) $f^{-1}(x)$

5) Solve for all values of x on the interval $[0, 2\pi)$

a) $\sin 2x + \sin x = 0$

b) $4\sin^2 x - 3 = 0$

c) $\cos^2 x - 3\cos x + 2 = 0$

d) $\cos 2x + \sin^2 x = 1$

6a) If $\sin x = \frac{3}{8}$ and $\cos y = -\frac{7}{8}$, with $\frac{\pi}{2} \le x \le \pi$ and $\pi \le y \le \frac{3\pi}{2}$, find $\sin(x - y)$ and $\tan(x - y)$

b) If $\cos x = \frac{6}{11}$ and $\tan y = -\frac{8}{11}$, with $\frac{3\pi}{2} \le x \le 2\pi$ and $\frac{\pi}{2} \le y \le \pi$, find $\sin(x + y)$ and $\cos 2y$

c) If $\tan W = \frac{5}{7}$ and $\cos Z = -\frac{2}{9}$, with $0 \le W \le \frac{\pi}{2}$ and $\frac{\pi}{2} \le Z \le \pi$, find $\cos(W + Z)$ and $\tan(2W)$

7) Given the function $f(x)$, find (a) the domain; (b) the value that $f(x)$ approaches as *x* approaches infinity; (c) at what value(s) does $f(x)$ cross the *x*-axis; and (d) at what value does $f(x)$ cross the *y*-axis?

a) $f(x)=\dfrac{5x^2-25x-30}{x^2+3x-28}$

b) $f(x)=\dfrac{x^2-16}{4x^2-4x-24}$

c) $f(x)=\dfrac{2x^2-2x-144}{3x^2-18x-120}$

8a) You initially have 800 grams of an element. Ten hours later you have 650 grams. What is its half-life?

b) You initially have 12 grams of an element. Two days later you have 1 grams. What is its half-life?

c) You initially have 70 grams of an element. Ten minutes later you have 10 grams. What is its half-life?

9a) Ship *A* sails from port at a speed of 12 miles per hour. Ship *B* leaves at the same time from the same port at a speed of 16 miles per hour. If the angle between them is $\dfrac{\pi}{3}$, how far apart are they after 10 hours?

b) Ship *A* sails from port at a speed of 20 miles per hour. Ship *B* leaves 30 minutes later from the same port at a speed of 30 miles per hour. If the angle between them is $\dfrac{\pi}{3}$, how far apart are they 4 hours after *A* left?

c) Person *A* leaves school and walks at a speed of 4 miles per hour. Person *B* leaves one hour later from the same school at a speed of 3 miles per hour. If the angle between them is $\frac{\pi}{4}$, how far apart are they 2 hours after *B* has left?

d) Bird *A* leaves its nest and flies at a speed of 80 miles per hour. Bird *B* leaves at the same time from the same nest at a speed of 100 miles per hour. If the angle between them is $\frac{2\pi}{3}$, how far apart are they after 15 minutes?