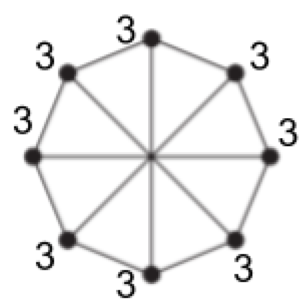
MAT 118 Spring 2017 Name ______________________________

Midterm #2 ID# ______________________________

Please show your work.
And remember: no calculators!
The test is out of 100 points.

1. [15 pts] Consider the following graph:

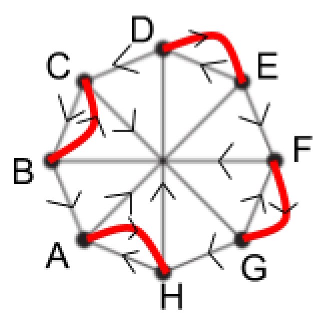


(a) Write the degrees of the vertices on the graph above. See above.

(b) Does this graph have any Euler circuits? Why or why not? No, it does not: it has odd vertices, so by Euler's Circuit Theorem, there are no euler circuits.

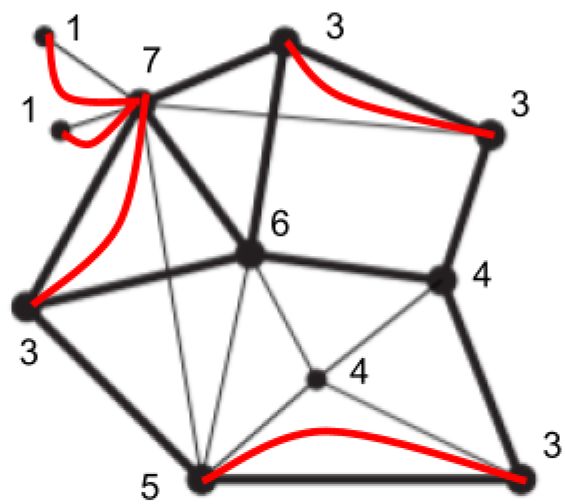
(c) Does this graph have any Euler paths? Why or why not? No, it does not: it has more than 2 odd vertices, so by Euler's Path Theorem, there are no euler paths.

(d) On the copy of the graph below, give an optimal Eulerization of the graph.



(e) Finally, on your eulerized graph, give an Euler circuit. AEDEFGFBCGHAHDCBA (see above)

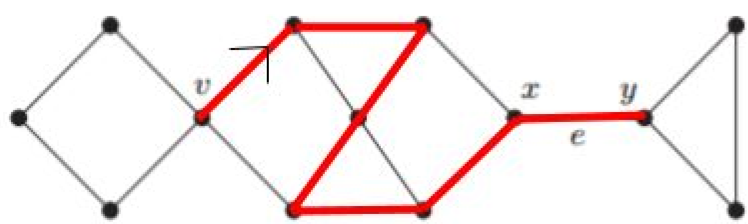
2. [10 pts] Consider the following graph (ignore the fact that some edges are thicker than others):



(a) Write the degrees of the vertices on the graph above. See above.

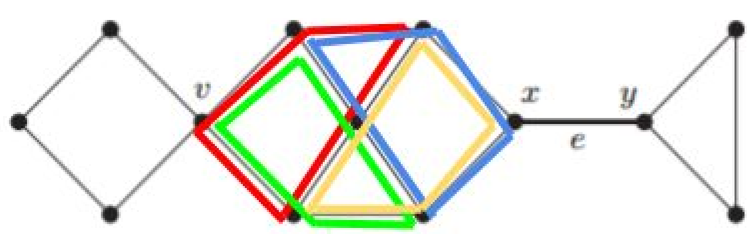
(b) On the same graph given above, draw an Eulerization of the graph. See above.

3. [10 pts] Consider the following graph:



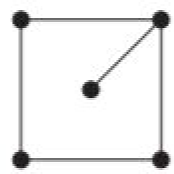
(a) Find a path from v to y of length 7. See above.

(b) Find all circuits in the graph of length 5; indicate where they are. How many are there?
There are 4 if we ignore directions (see below), and 8 if we care about directions.

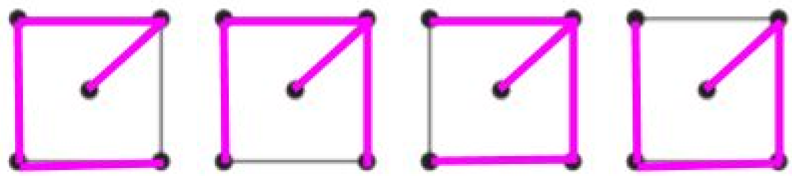


(c) Is edge "e" a bridge? Yes, since removing *e* leaves a disconnected graph.

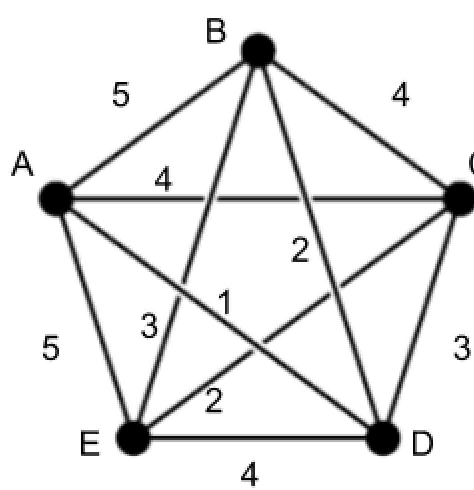
4. [10 pts] Find all spanning trees of the graph below. Draw a separate graph for each.



There are 4 spanning trees:



5. [15 pts] Consider the following weighted graph:



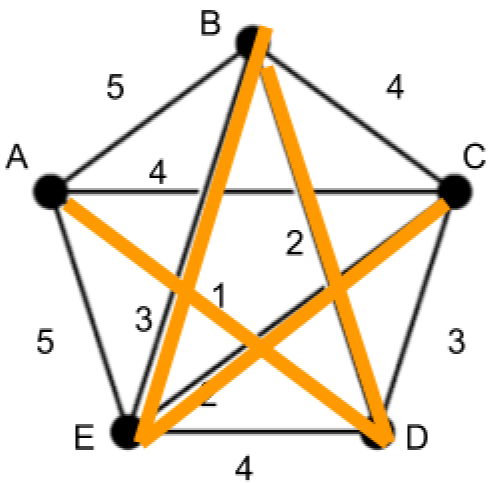
(a) Using the NNA starting at A, give the resulting Hamilton circuit and its total weight.
Hamilton circuit = ADBECA, total weight = 1 + 2 + 3 + 2 + 4 = 12.

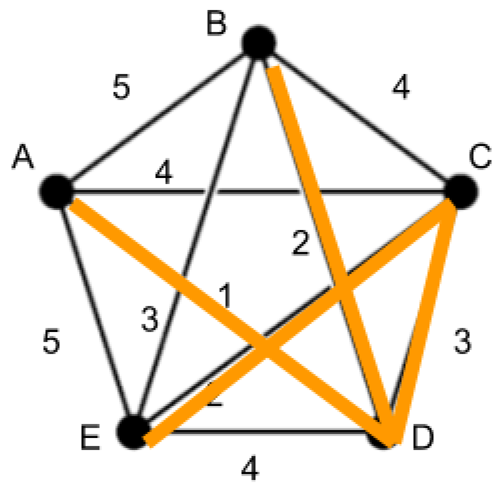
(b) Using the NNA starting at E, give the resulting Hamilton circuit and its total weight.
Hamilton circuit = ECDABE, total weight = 2 + 3 + 1 + 5 + 3 = 14.

(c) Using the Cheapest link algorithm, give the resulting Hamilton circuit and its total weight.
We get the same circuit as in (a), with total weight 12.

6. [10 pts] For the weighted graph in problem #6, reproduced below, use Kruskal's algorithm to find an MST, and give its total weight.

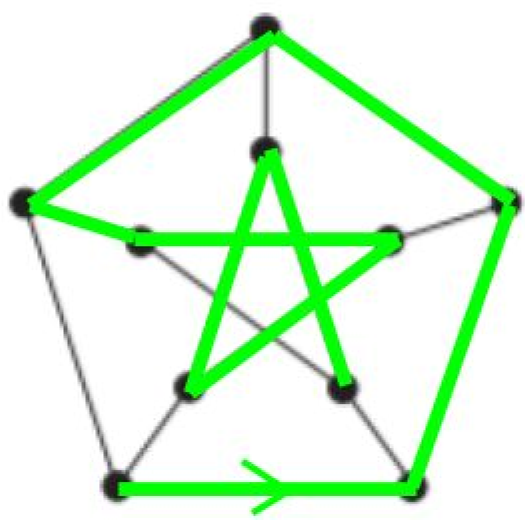
There are 2 answers, both of total weight 1 + 2 + 2 + 3 = 8:





7. [10 pts] Find a Hamilton path in the graph below.

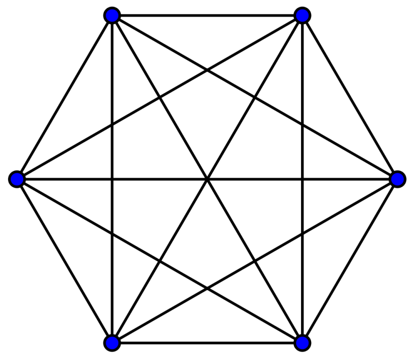
Here's one (there are many possibilities):



8. [15 pts]

(a) Draw a K_6 graph below.

Remember that there are many ways to draw this. The defining property of this graph, which must be apparent in any pictorial representation, is that any pair of distinct vertices has exactly one edge connecting them. Here is a picture:



(b) If each edge has weight 5, what is the total weight of an MST in your K_6 graph?

There are 6 vertices in K_6. Any spanning tree has 5 edges, by the N-1 edges property. Thus an MST, which is any spanning tree, since the edges all have the same weights, has total weight equal to $5 \times 5 = 25$.

(c) If instead one edge has weight 2, and the rest have weight 5, what is the total weight of an optimal Hamilton circuit in your K_6 graph?

Any Hamilton circuit in this graph will have 6 edges. To be optimal, the edge of weight 2 must be among these, and the remaining 5 edges will be of weight 5. Thus the total weight of an optimal Hamilton circuit is $2 + 5 \times 5 = 27$.

9. [5 pts] How many spanning trees does the following graph have? One! It's a tree.