MAT 118 **Final Exam** **December, 2002**

Name: _____ **Id & Section** _____

Section number is your **recitation** number, e.g. 01, 02, etc.

DO NOT OPEN THIS EXAM UNTIL INSTRUCTED

You have 2 hours and 30 minutes to complete this exam. You may use a calculator. You may **NOT** use any books or notes.

Please answer each question in the space provided. If you run out of room for answer continue of the back of the page. Please SHOW YOUR WORK and EXPLAIN YOUR REASONING wherever possible. Please write full **solutions**, not just answers. Unless otherwise marked, **answers without justification will get little or no partial credit**. Cross out anything the grader should ignore and circle or box the final answer.

This final has 7 questions, for a total of 250 points. Good luck!

You may need to use the following formulas and pictures:

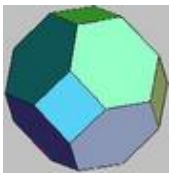
$$F = P\Big(1 + \frac{r}{n}\Big)^{nt}$$ F = future value after t years, P = principal, r = annual interest rate, n = number of periods per year

$$F = D\Big(\frac{(1 + \frac{r}{n})^{nt} - 1}{\frac{r}{n}}\Big)$$ F = future value after t years, D = deposit, r = annual interest rate, n = number of periods per year

$$P\Big(1 + \frac{r}{n}\Big)^{nt} = R\Big(\frac{(1 + \frac{r}{n})^{nt} - 1}{\frac{r}{n}}\Big)$$ P = principal, R = amount of each payment, r = annual interest rate, n = number of periods per year

$$_nC_r = \frac{n!}{r!(n-r)!}, \qquad n! = n(n-1)\cdots 1$$

Octahedron/truncated octahedron:



Problem	1	2	3	4	5	6	7	**Total:**
Max	40	45	20	70	25	25	25	250
Scores								

1. For an upcoming election involving candidates Alice, Bob, and Chris, a preliminary poll is taken and the following voter preference rankings are collected.

	9	28	11	23	23	6
Alice	1	1	2	3	2	3
Bob	2	3	1	1	3	2
Chris	3	2	3	2	1	1

Use the preference rankings collected in the poll to give answers to the following questions:

(a) (10 points) Which candidate will win using the plurality method?

(b) (10 points) Which candidate will win using the plurality method followed by a runoff between the top two finishers? Show your work.

(c) (10 points) Which candidate will win using Borda's Method? Show your work.

(d) (10 points) Which candidate (if any) is a Condorcet winner? Show your work.

2. (a) (20 points) If John takes out a loan of $50,000 at 5% interest compounded monthly to be paid back by equal payments at the end of each month for 20 years, how much will John have to pay each month?

(b) (10 points) How much interest will John pay over the life of the loan?

(c) (15 points) If John's lender deposits $100 out of each of his loan payments as he receives them during the 20 year period (in an account earning 6% interest compounded monthly), how much will he have in that account at the end of the 20 years?

3. (20 points) Mrs. Kelly's first grade class has 10 children. Five of them are boys and five are girls. Mrs. Kelly randomly chooses 6 of these 10 children to be in her class play. What is the probability that exactly 3 of these 6 kids chosen for the play are boys?
(Hint: Describe the sample space and count with the help of the formula for combinations on the cover page)

4. The following table shows distances in miles between four villages named A, B, C, D:

	A	B	C	D
A		5	9	6
B	5		3	2
C	9	3		5
D	6	2	5	

(a) (10 points) Sketch a weighted graph representing these four villages and the distance between them.

(b) (15 points) Use the *Nearest neighbor algorithm* (starting with vertex A) to find a Hamiltonian circuit. What is the length of the circuit?

(c) (15 points) Use the *Greedy algorithm* to find a Hamiltonian circuit. What is it's length?

(d) (15 points) Does any of the two algorithms provide the exact solution for the *Travelling salesman problem* for this graph? Justify your answer!

(e) (15 points) A company having one office in each of the above four villages needs to provide a computer network between them. Suppose that the monthly cost of leasing a computer line between any of the four villages is proportional to the distance between them. What is the most economical solution for the company to network all its offices? If the monthly leasing cost of the line is $10 per mile, what is the total monthly cost of this network?

5. (a) (15 points) Find the measure of the angle of a regular 11-gon.

   (b) (10 points) Explain why is it impossible to construct a semiregular monohedral tiling that about each vertex has (at least) two regular 11-gons.

6. (a) (10 points) Find the number of vertices, edges and faces of the octahedron.

   (b) (15 points) A truncated octahedron is the semiregular solid obtained by uniformly slicing off the corners of the octahedron so that the resulting faces are regular polygons. What kind of faces does the truncated octahedron have and how many there are of each type ?

7. Decide whether each of the following statements is true or false:

   (a) (5 points) Every election has a Condorcet winner.

   (b) (5 points) A tree is a connected graph with no circuits.

   (c) (5 points) There is a regular tiling of the plane with pentagons.

   (d) (5 points) We can always find an exact solution to the Travelling salesman problem if we just do the *Nearest neighbor alogorithm* repeatedly, one time starting with each vertex, and then pick the shortest circuit produced.

   (e) (5 points) The dual of the cube is the icosahedron.