EXAM

Fianl

Math 118

December 19, 2001

ANSWERS

Problem 1. A woman deposits \$800 into a savings account earning 7.2% interest compounded quarterly. How long will it take for the account to reach \$2000?

Answer:
We use $F = P\left(1+\frac{r}{n}\right)^{nt}$:

$$\begin{aligned}
\$2000 &= \$800\left(1+\frac{.072}{4}\right)^{4t} \\
\frac{2000}{800} &= \left(1+\frac{.072}{4}\right)^{4t} \\
\log\frac{2000}{800} &= \log\left(\left(1+\frac{.072}{4}\right)^{4t}\right) \\
\log\frac{2000}{800} &= 4t\log\left(1+\frac{.072}{12}\right) \\
0.39794001 &= (4)(0.007747778)t \\
t &= 12.84.
\end{aligned}$$

Problem 2. It is possible, but not easy, to draw a three-regular, diameter three graph with twelve vertices on the plane without crossings. How many faces will such a drawing have?

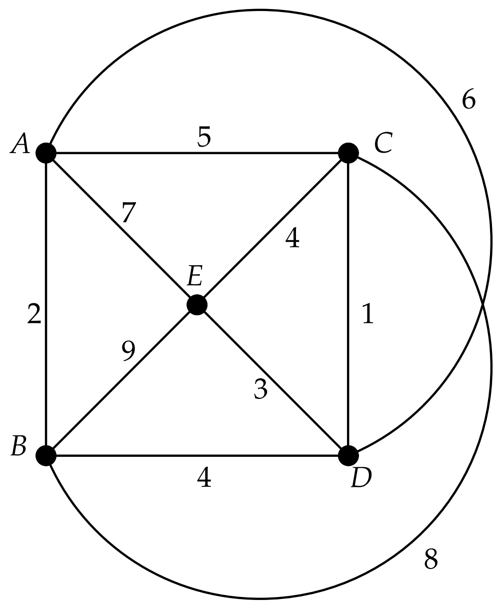
Answer:
Any three regular graph with 12 vertices will have 18 edges. Therefore, using the fact that for any planar graph we have

$$v - e + f = 1,$$

we substitute $v = 12$ and $e = 18$ to get

$$12 - 18 + f = 1 \Rightarrow f = 7.$$

Problem 3. Consider the following weighted graph and the travelling salesman problem for this graph:

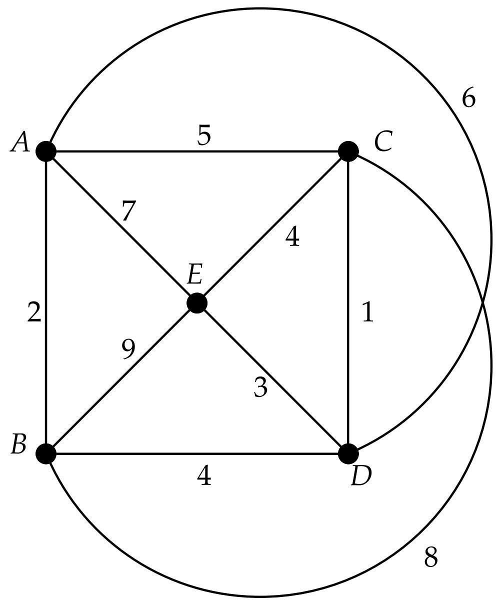


(a) On the picture above, carefully sketch the Hamiltonian circuit produced by the nearest neighbor algorithm using C as the starting vertex. Indicate its length.

Problem 4.
The Hamiltonian path produced is $CDEABC$ and has length $1+3+7+2+8=21$.

Problem 4. *Continued.*

Here is another picture of the same graph:



(b) On this picture, carefully sketch the Hamiltonian circuit produced by the greedy algorithm. Indicate its length.

We choose the edges CD, AB, DE, AC, BE to get the Hamiltonian path $ABEDCA$ with length $2+9+3+1+5=20$.

Problem 5. Circle the one that yields the largest account.

- Making monthly deposits of \$100 at 16% interest for 12 years.

- Making monthly deposits of \$200 at 8% interest for 12 years.

- $\boxed{\text{Making monthly deposits of \$100 at 8\% interest for 24 years.}}$

Answer:
A little experience with these problems suggest that last option is definitely the one that yields the greatest account, but you can check the value of each using:

$$F = D\left(\frac{\left(1+\frac{r}{n}\right)^{tn}-1}{\frac{r}{n}}\right)$$

Problem 6. A school club with 25 members is voting on whether to play soccer, baseball, or football at their club picnic. The following preference rankings were collected:

	4	3	5	3	4	6
Soccer	1	1	2	3	2	3
Baseball	2	3	1	1	3	2
Football	3	2	3	2	1	1

(a) Which game will win if the decision is made by plurality with a runoff between the top two choices?

Answer:

Football gets 10 votes, Baseball gets 8 and Soccer gets 7, so the runoff will be between Football and Baseball. In the runoff election, we see that

	4	3	5	3	4	6
Baseball	2	3	1	1	3	2
Football	3	2	3	2	1	1

13 voters prefer Football and 12 voters prefer Baseball. So, Football wins if the decision is made by plurality with a runoff.

(b) Which game will win if the decision is made by Borda's method?

Answer:

Here are the Borda counts:

Soccer: $7 \times 3 + 9 \times 2 + 9 \times 1 = 47$
Baseball: $8 \times 3 + 10 \times 2 + 7 \times 1 = 51$
Football: $10 \times 3 + 6 \times 2 + 9 \times 1 = 51$.

Borda's method results in a tie between Football and Baseball.

Problem 6. *Continued.*

(d) Suppose that the decision will be made using the simple plurality method. Which groups of voters can vote strategically in order to bring about a preferencial outcome? Circle them:

- The four voters who ranked soccer first, baseball second, and football last.
- The five voters who ranked baseball first, soccer second, and football last.
- The four voters who ranked football first, soccer second, and baseball last.
- The three voters who ranked soccer first, football second, and baseball last.
- The three voters who ranked baseball first, football second, and soccer last.

Answer:
If the voters in either of the first two groups vote for their second choices then their second choices will win, which is preferable to Football, which is their third choice.
The voters that want Football to win, cannot do any better than vote as they already have, for Football, and bring about the victory of their first choice.
The remaining voters could influence the outcome of the election by voting for their last choice instead of their first choice, but this would not be preferencial since they all prefer football to their last choice.

Problem 7. Use the Euclidean algorithm to find the greatest common divisor of 385 and 88. Then, find integers x and y so that $\gcd(385, 88) = x \cdot 385 + y \cdot 88$.

Answer:

$$\begin{aligned}
385 &= 4 \times 88 + 33\\
88 &= 2 \times 33 + 22\\
33 &= 1 \times 22 + 11\\
22 &= 2 \times 11 + 0.
\end{aligned}$$

So, $\gcd(385, 88) = 11$. So, we can write $11 = x \cdot 385 + y \cdot 88$ and we do so by recycling the computations above. We first rewrite the first three equations above as:

$$\begin{aligned}
33 &= 385 - 4 \times 88\\
22 &= 88 - 2 \times 33\\
11 &= 33 - 1 \times 22.
\end{aligned}$$

Starting with the last, and substituting, we get:

$$\begin{aligned}
11 &= 33 - 1 \times 22\\
&= 33 - 1 \times (88 - 2 \times 33)\\
&= -1 \times 88 + 3 \times 33\\
&= -1 \times 88 + 3 \times (385 - 4 \times 88)\\
&= 3 \times 385 - 13 \times 88
\end{aligned}$$

So, if $x = 3$ and $y = -13$ then we have $11 = x \cdot 385 + y \cdot 88$.

Problem 8. True or false:

(a) The graph $K_{10,6}$ has 16 vertices and 60 edges.

Answer:

True

(b) If every vertex of a graph has even degree, then that graph has an Euler circuit.

Answer:

True

(c) Every election has a Condorcet winner.

Answer:

False

(d) 10619 is prime.

Answer:

False

(e) We can always find an exact solution to the travelling salesman problem if we just do the nearest neighbor alogorithm repeatedly, one time starting with each vertex, and then pick the shortest circuit produced.

Answer:

False