

How can one simplify the following?

$$\frac{9x^4 + 6x^2 + 1}{x + 2}$$

In this case one can either use long division or synthetic division.

Long Division:

$$\begin{array}{r}
 9x^3 - 18x^2 + 42x - 84 \\
 x + 2 \overline{) 9x^4 + 6x^2 + 1} \\
 \underline{9x^4 + 18x^2} \\
 -18x^2 + 6x^2 + 1 \\
 \underline{-18x^2 - 36x^2} \\
 42x^2 + 1 \\
 \underline{42x^2 + 84x} \\
 -84x + 1 \\
 \underline{-84x - 168} \\
 169
 \end{array}$$

Answer: $9x^3 - 18x^2 + 42x - 84$ with a remainder of 169.

Synthetic Division:

- special case of long division when the divisor is a linear factor

$$\begin{array}{r|rrrrr}
 -2 & 9 & 0 & 6 & 0 & 1 \\
 & \downarrow & -18 & 36 & -84 & 168 \\
 \hline
 & 9 & -18 & 42 & -84 & 169
 \end{array}$$

Answer: $9x^3 - 18x^2 + 42x - 84$ with a remainder of 169.

Most of the time when synthetic division is introduced to students, they are told here is how it works and so just follow all the steps and you will get the answer. The students do not understand why synthetic division works or where it comes from, they are just able to compute it. So how can we as future teachers be capable to explain synthetic division to our students and related it to long division?

The numbers in red represent the connection from long division to synthetic division. When teaching, one can explain to the students the corresponding numbers in this manner.

Let's try another example
Simplify:

$$\frac{2x^4 - x^2 + x + 1}{x^2 - 4}$$

Long Division:

$$\begin{array}{r}
 2x^2 + 7 \\
 x^2 - 4 \overline{) 2x^4 - x^2 + x + 1} \\
 \underline{2x^4 - 8x^2} \\
 7x^2 + x + 1 \\
 \underline{7x^2 - 28} \\
 x + 29
 \end{array}$$

Answer: $2x^2 + 7$ with a remainder of $x + 29$.

Synthetic Division: Since we are dividing by $x^2 - 4$ we need to consider that by shift everything over as shown below.

$$\begin{array}{r|rrrrr}
 4 & 2 & 0 & -1 & 1 & 1 \\
 & \downarrow & & 8 & & 28 \\
 \hline
 & 2 & 0 & 7 & 1 & 29
 \end{array}$$

Answer: $2x^2 + 7$ with a remainder of $x + 29$.

Now let's try to simplify the following as we did previously:

$$\frac{2x^4 + x^3 - x^2 + x + 1}{x^2 - 4}$$

Long division:

$$\begin{array}{r}
 2x^2 + x + 7 \\
 x^2 - 4 \overline{) 2x^4 + x^3 - x^2 + x + 1} \\
 \underline{2x^4 - 8x^2} \\
 x^3 + 7x^2 + x \\
 \underline{x^3 - 4x} \\
 7x^3 + 5x \\
 \underline{7x^3 - 28} \\
 5x + 29
 \end{array}$$

Answer: $2x^2 + x + 7$ with a remainder of $5x + 29$.

Synthetic division:

$$\begin{array}{r|rrrrr}
 4 & 2 & 1 & -1 & 1 & 1 \\
 & \downarrow & & 8 & & 28 \\
 \hline
 & 2 & 1 & 7 & 1 & 29
 \end{array}$$

Answer: $2x^2 + x + 7$ with a remainder of $5x + 29$.

Homework:

Simplify the following using both long division and synthetic division, state how synthetic division would work in this case.

$$\frac{2x^4+x^3+x^2+1}{x^2-x+4}$$