Class Notes from March 26th

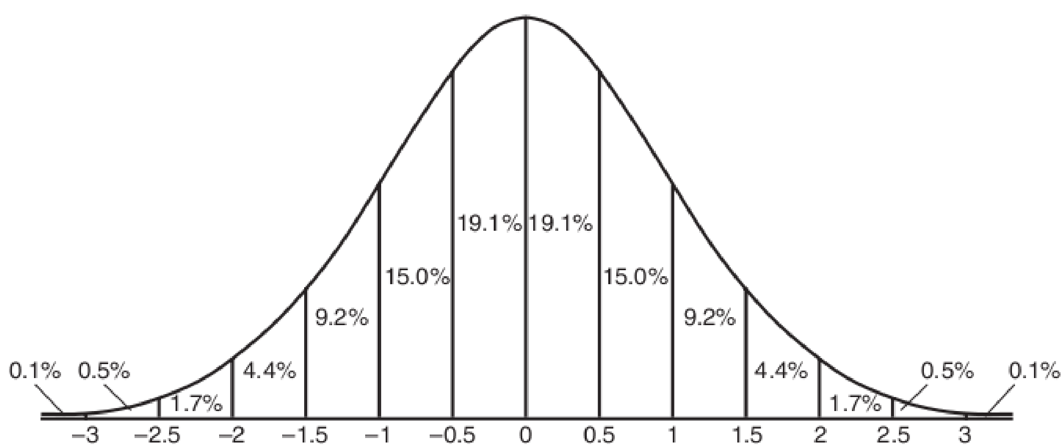
Today in class we discussed the in-class Math B Regents Exam. The next class (March 31st) will be the retake of the Math B Regents. This class will be spent going over practice regents questions, and discussing specific test questions from the first regents taken in class on March 5th.

From the in-class Math B Regents Exam taken on March 5th:

5. On a standardized test, the distribution of scores is normal, the mean of the scores is 75 and the standard deviation is 5.8. If a student scored 83, the student's score ranks
    1. below the 75th percentile
    2. between the 75th percentile and the 84th percentile
    3. between the 84th percentile and the 97th percentile
    4. above the 97th percentile

Solution: Here we are given the mean (average) of the scores, the standard deviation (σ) and the score of a particular student. So:

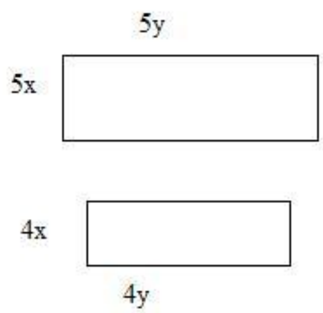
Mean = 75
Standard Deviation = σ = 5.8
Student's score = 83



Now we must look at the Normal Curve on the given formula sheet. Since we know the mean score 75, we can translate the standard normal curve so the mean moves from 0 to 75. This mean corresponds to the 50th percentile. Adding one standard deviation will give a score in the 50 + 19.1 + 15 = 84.1th percentile. This percentile corresponds to a score of 75 + 5.8 = 80.8 on our exam. Two standard deviations above the mean will be the 84.1 + 9.2 + 4.4 = 97.7th percentile; this corresponds to a score of 75 + 5.8 + 5.8 = 86.6. Clearly $80.8 < 83 < 86.6$ so the student's score lies between one and two standard deviations above the mean. According to the graph, the range of these scores lies between the 84.1th and 97.7th percentile. Thus, the correct answer is choice (3).

12. The lengths of the sides of two similar rectangular billboards are in the ration 5:4. If 250 square feet of material is needed to cover the larger billboard, how much material in square feet, is needed to cover the smaller billboard?

Solution: Here we are dealing with similar rectangles, with side lengths in the ratio of 5:4. The larger rectangle has an area of 250. What we are looking for is the area of the smaller rectangle. The dimensions of the two rectangles are shown below.



So now we know that since the top rectangle is the larger rectangle:

$$\text{Area} = (5x)*(5y) = 25xy = 250$$
$$\text{Thus:} \quad xy = 10$$

Now we can look at the smaller rectangle.

$$\text{Area} = (4x)(4y) = 16xy = 16(10) = 160$$

Thus 160 square feet of material is needed to cover the smaller billboard.

13. In a certain school district, the ages of all new teachers hired during the last 5 years are normally distributed. Within this curve, 95.4% of the ages, centered about the mean, are between 24.6 and 37.4 years. Find the mean age and the standard deviation of the data.

Solution: Here we know that 95.4% of the ages lie in the following interval: $24.6 \le x \le 37.4$. As before, we use the Normal Curve from the formula sheet (or the previous page). By adding the percentages starting at 0 (where the mean lies)) we can see that in a standard normal curve, 95.4% of the samples lie within -2σ and 2σ. Since we are told that in this school district, 95.4% of the age ranges from 24.6 to 37.4, we can use that to calculate the mean and standard deviation.
First, the mean will be the average of the two values: $\mu = (37.4 + 24.6)/2 = 31$. **The mean age of the teachers is 31**.
In order to calculate the standard deviation, we observe that $37.4 – 31 = 6.4$, which corresponds to 2σ. Thus, **the standard deviation of ages is 3.2**.
As a check, we can confirm that $24.6 = 31 - 2*(3.2)$.

14. After studying a couple's family history, a doctor determines that the probability of any child born to this couple having a gene for disease X is 1 out of 4. If the couple has three children, what is the probability that exactly two of the children have the gene for disease X?

Solution:
To figure this out, note that the probability of one child having the gene is ¼, and the probability of not having the gene is ¾. Thus, the probability that the two youngest children have the gene and the oldest does not will be ¼* ¼ * ¾ . However, there are several other ways that exactly two children can have the gene (the one without the gene could be the youngest, the middle child, or the oldest). Thus, the probability that this can occur will be 3*(¼)(¼)(¾) = 9/64.

19. Solve for x: $\log_4(x^2 + 3x) - \log_4(x+5) = 1$

Solution:
We must solve for x. We recall the rules we know for working with logarithms. Using the rule we know for subtraction of logs, we can change this equation to the following:

$$\log_4[\ (x^2 + 3x) / (x+5)\ ] = 1$$

Now we just have one log equal to a constant. Recall that $\log_4 4 = 1$, so the equation can be written as

$$\log_4[\ (x^2 + 3x) / (x+5)\ ] = \log_4 4$$

Raising 4 to each side gives

$$(x^2 + 3x) / (x+5) = 4$$

Or, as long as $x \neq -5$,

$$x^2 + 3x = 4x + 20$$
$$x^2 - x - 20 = 0$$
$$(x-5)(x+4) = 0$$

Thus either x=5 or x=-4.

It is always a good idea to check our answers. If x=5, the original equation says that $\log_4[\ (25 + 15) / (10)\ ] = 1$, which is certainly true since $\log_4[\ 4\ ] = 1$.
If x=-4, we get $\log_4[\ (16-12) / 1\ ] = 1$, which is also true.

We looked at about 10 other Regents problems, some easy, some less so. But it seems no notes were taken on those.