MAE301

3/17

*Regents retake on Tuesday, 3/31.

Relations: What are some of the relations of interest to us?

- functions
- equivalence relations (which lead to partitions)
- order

If you have an equivalence relation on $\mathbb{X}$ then you can form a partition of $\mathbb{X}$ into equivalence classes.

$$[a] = \{x \epsilon \mathbb{X} | x \sim a\}$$

$$\mathbb{X}_{/\sim} = \{[a] | a \epsilon \mathbb{X}\}$$

This is what we've already been doing to build up to the set of Reals from the Naturals:

$$\mathbb{N}$$

$$\mathbb{N}_{/\sim 1} = \mathbb{Z}$$

$$\mathbb{Z}_{/\sim 2} = \mathbb{Q}$$

$$\left(\prod_{i=1}^{\infty} \mathbb{Z}_{10}\right)_{/\sim 3} = \left(\prod_{i=1}^{\infty} \mathbb{Q}\right)_{/\sim 4} = \mathbb{R}$$

<u>Functions</u>:

- In elementary school there are numbers and operations (using numbers to get more numbers).
  - Any symbols are like constants (such as the area of a square: $A = s^2$)
- In Jr. High to (early) High school variables come into play.
  - Now symbols can be variables or constants (ex: $y = 3x + 2$). However there is an issue of how to calculate vs. a relation.

- In senior High School through college (starting roughly with precalculus) functions become mappings.

**Even though there is this increase in abstraction we must keep in mind that some students at the higher levels might still be thinking in the earlier ways.*

When is a function $f: A \to B$ invertible?

(Note that this is mildly different from: "given a fuction $f(x) = \frac{3x+2}{5x-4}$, is it invertible?")

f is invertible if it is one to one. (It doesn't necessarily matter if it is onto since we can restrict the domain of f^{-1} to be the image of f. $\{y|f(x) = y\ for\ x\epsilon dom(f)\}$

So given $f(x) = \frac{3x+2}{5x-4}$

Find $f^{-1}(x)$

Process: let $y = f(x)$, switch x and y,

$$x = f(y)$$

Solve for y.

Graphically, the inverse is a reflection in the line $y = x$.

Students need to understand that the step to switch x and y, $x = f(y)$, is purely notational.

$y = \frac{3x+2}{5x-4}$ $5xy - 4y = 3x + 2$ $x(5y-3) = 2 + 4y$ $x = \frac{2+4y}{5y-3} = f^{-1}(y)$ $y = \frac{2+4x}{5x-3} = f^{-1}(x)$ With an implicit $x \neq \frac{4}{5}$ on $f(x)$ and $y \neq \frac{3}{5}$ on $f(y)$	Students will complain that this is wrong because you didn't switch *x* & *y* first. Since they don't fully understand the concepts, they just want to follow steps.

Similarly, for $y = x^2$, without really saying it we restrict $f(x)$ to where it is one to one and solve.

$$x = y^2$$

A:	**B:**	**A** and **B** unified:
Think: $y > 0$ AND $x > 0$ $\sqrt{x} = \sqrt{y^2}$ $y = \sqrt{x}$	Also: $x > 0$ AND $y < 0$ $\sqrt{x} = \sqrt{y^2} = -y$ $y = -\sqrt{x}$	$y = \pm\sqrt{x}$ *This is **not** a function. It's shorthand for two functions.

Ideally $\pm$ would not be used. It would be written as two parts so students understand you're considering two possibilities.

Generation definition of a function:

A=Domain=$\{1,2,3\}$

B=Co-Domain=$\{4,5,6\}$

How many functions are there from A→B?

The answer is $3 * 3 * 3 = 27$

What about invertible functions?

$f(1) =$ {Choose 1 of the 3 in B}

$f(2) =$ {Choose 1 of the remaining 2 in B}

$f(3) =$ {Choose the last one in B}

This is $3*2*1=6$ invertible functions.

What about for $f:\{1,2,3\} \to \{4,5,6,7\}$

There are 4^3 functions with $4*3*2$ invertible.

A function is completely determined by its input and output

The question of functions can be recast in the following way:

"How many permutations are there of $\{4,5,6,7\}$ taking 3 at a time?"

Rephrase and ask ourselves:

"How many ways can we choose 3 things out of a set of 4?"

$$\binom{4}{3}$$

This is the number of permutations of 3 out of 4 divided by the number of permutations of those 3 things.

In general: $\binom{n}{k} = \frac{n!}{k!(n-k)!}$

Where $\frac{n!}{(n-k)!}$ is the permutation part

And $\frac{1}{k!}$ is the arrangement part

In Summary:

- Ordering n objects out of k with repetition allowed is n^k
- Ordering n objects out of k without repetition is $\frac{k!}{(k-n)!}$ (Permutation ${}_kP_n$)
- Choose n out of k objects without repetition or order is $\frac{n!}{k!(n-k)!}$ (Combination $\binom{n}{k}$)