Chapter 1

Preliminaries about Maple

Maple is a comprehensive general purpose computer algebra system which can do symbolic and numerical calculations and has facilities for 2- and 3-dimensional graphical output. Calculus courses may be structured so that Maple can be used as a tool that will help you gain a complete understanding of the material discussed. You will discover that the capability of Maple goes well beyond the realm of calculus; it is a tool that is used more and more in education and scientific research in mathematics and engineering.

This chapter is a brief introduction to the use of Maple in the environment that you will discover at Stony Brook. It is not exhaustive and its sole purpose is to guide you through the first steps needed to start using this tool. Once you become familiar with the basics of Maple and can appreciate its advantages, we encourage you to experiment with it, not only to learn mathematics in more detail but also to help you in other courses and your own research. More advanced documentation is available on line, in the computer lab, and in the library.

Maple runs on many different types of computers and operating systems. For example, you might use Maple from a Sun workstation running Solaris (a flavor of Unix) in the computer lab, from a Windows computer in the library, and from a Macintosh at home. Maple behaves similarly on all of them; we will usually point it out when we use something specific to one type of computer.

1 Starting a Maple Session

How to start a Maple session varies somewhat in different enviroments. On the Windows machines, you can usually either start Maple by selecting the option from the start menu, or by finding the appropriate icon (it usually looks like a Maple leaf) and clicking the mouse on it. In a Unix environment, there may be an icon or menu item for Maple; you can usually always just issue the command `xmaple &` or `maple -x &` to start it.

After a brief pause, you should see a window that looks similar to that in Figure 1; this is Maple's graphical (or worksheet) interface, where you will type your commands and see the

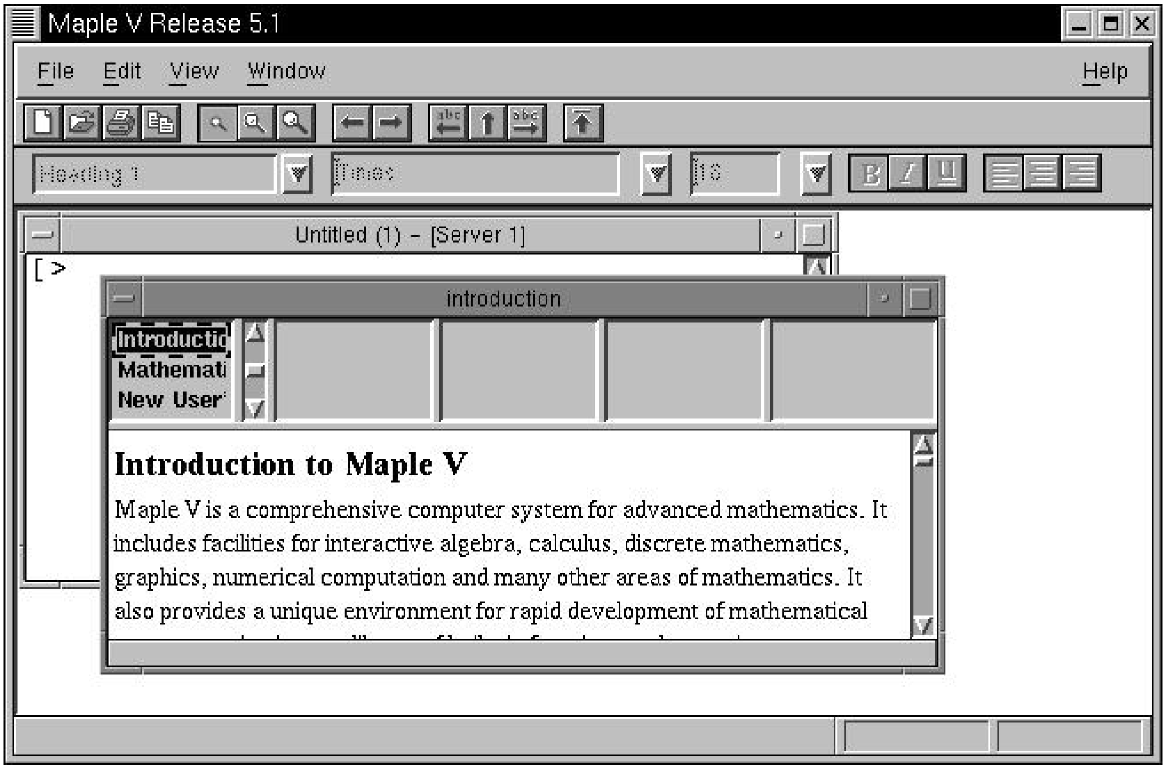


Figure 1: A newly-opened Maple worksheet.

results. We will go into more details of the worksheet interface in §3. As in the figure, you're session may have started with two windows open inside the main Maple window. The one in front in Figure 1 is a help window; we'll come back to this in §3.3. But for now, let's concentrate on a worksheet where we can enter some Maple commands. The top part of this window should look something[1] like this:

This window is a Maple worksheet, and is a place to enter your Maple commands. For now, click on the button which maximizes the current worksheet; this is on right side of the worksheet's title bar, and looks like ▢.[2] Clicking the mouse on this should make the worksheet fill most of the Maple window.

Most users typically use the Maple's graphical, or worksheet, interface; this is what starts you add the `-x` to the `maple` command in Unix. However, there is also a text-based interface, which you might see if you just type `maple`, omitting the `-x`.

[1] This particular image was made on a computer running Unix. Under Microsoft Windows or on a Macintosh, the symbols around at the edges will be slightly different, but with similar functions.

[2] Again, if you are running another type of computer, the button may look a little different. For example, under MS-Windows, it looks similar to this: ▣.

```
    |\^/|     Maple V Release 5.1 (SUNY Stony Brook)
._|\|   |/|_. Copyright (c) 1981-1998 by Waterloo Maple Inc. All rights
 \  MAPLE  /  reserved. Maple and Maple V are registered trademarks of
 <____ ____>  Waterloo Maple Inc.
      |       Type ? for help.
>
```

This interface is useful when you want to run Maple in a non-graphic environment; for example, over a dial-up connection. However, keep in mind that while all the usual Maple commands work, this interface is much more limited— it is difficult to edit your commands or save your work, and graphic commands such as `plot` are much more limited. If you get into this mode by mistake, you can just type the Maple command `quit;` to exit.

2 Basic Maple

You can use Maple for numerical calculations, for symbolic manipulations, and for graphing purposes. *All Maple commands must be ended with a semicolon* ; *and the corresponding stroke of the return key.* To familiarize yourself with it, try to run the following examples and analyze the results. The commands that you type in and the Maple results are shown here in a way that is similar to what you see on the screen.

2.1 Maple for numeric calculations

Most of the time you will be using Maple as a calculator. You can also write programs in Maple, but usually you will be using the tool expecting an immediate response. You type in your statement at the Maple prompt `>` and Maple responds with an answer:

```
> 1/2+3;
```

$$\frac{7}{2}$$

The symbols for the basic operations are +, `-`, `*` and `/`. Exponentiation is denoted by `^`:

```
> 77*(3^5+5^2)/(99-47);
```

$$\frac{5159}{13}$$

In the first example, the inverse of 2 is added to 3. In the second, we input $\frac{77(3^5+5^2)}{99-47}$, and Maple computes its value before printing the output.

You must always use the asterisk `*` for multiplication. Forgetting it in expressions like `sin(2*x)` or `4*x` is easy, and may result in a syntax error message. Even if you don't get a syntax error, Maple will not treat the expression as you expect.

Maple works primarily with whole numbers or fractions. However, you can force it to write the decimal expansion of any number with `evalf`:

```
> evalf(100*Pi);
```

$$314.1592654$$

```
> b:=sqrt(2);
```

$$b := \sqrt{2}$$

```
> evalf(b);
```

$$1.414213562$$

```
> evalf(b,47);
```

$$1.4142135623730950488016887242096980785696718754$$

Be careful with this example. The *assignment* of the variable `b` to $\sqrt{2}$ is done with `:=` rather than with `=`. This will be the case of any other assignment operation you do in Maple, as we discuss below. The equals sign `=` is used to signify that two things are equal, not to set one to the other.

Notice the difference in the following two examples:

```
> sum(1/(2*i),i=1..10);
```

$$\frac{7381}{5040}$$

```
> sum(1.0/(2*i),i=1..10);
```

$$1.464484127$$

In both cases, we compute $\sum_{i=1}^{10}(2i)^{-1}$. However, in the second case, the presence of `1.0` indicates to Maple the terms in this computation are approximate, and gives you a decimal expansion of the result.

2.2 Maple for symbolic manipulations

We can ask Maple to give us a formula for the sum of the cube of the first n integers:

```
> sum(i^3,i=1..n);
```

$$\frac{1}{4}(n+1)^4 - \frac{1}{2}(n+1)^3 + \frac{1}{4}(n+1)^2$$

We can ask to factor the resulting polynomial in n:

```
> factor(%);
```

$$\frac{1}{4}n^2(n+1)^2$$

In this example, Maple's "ditto operator" `%` is used[3] to refer to the result previously obtained. Thus, in effect, the example above is equivalent to

```
> factor(1/4*(n + 1)^4 - 1/2*(n + 1)^3 + 1/4*(n + 1)^2);
```

$$\frac{1}{4}n^2(n+1)^2$$

One can expand or factor conveniently:

```
> p:=(x+1)^4*(x-6)^3;
```

$$p := (x+1)^4(x-6)^3$$

```
> expand(%);
```

$$x^7 - 14x^6 + 42x^5 + 112x^4 - 287x^3 - 882x^2 - 756x - 216$$

```
> factor(%);
```

$$(x+1)^4(x-6)^3$$

We can use Maple to solve equations (if `b` still has the value $\sqrt{2}$ from the previous section, first *unassign* it by executing the command `b:='b'`):

[3]In Maple V release 4 and earlier, Maple used the double-quote `"` to refer to the result of the previous command instead of `%`. If you encounter an older Maple program, you might have to change this, among some other things.

```
> quadeq:=a*x^2+b*x+c;
```

$$quadeq := ax^2 + bx + c$$

```
> solve(quadeq=0,x);
```

$$\frac{1}{2}\frac{-\sqrt{2}+\sqrt{2-4ac}}{a}, \frac{1}{2}\frac{-\sqrt{2}-\sqrt{2-4ac}}{a}$$

Notice that `solve` provides both roots of the quadratic polynomial.

Maple also divides polynomials. Here the command you execute has a syntax which is a little bit more complicated. Consider the following example:[4]

```
> p:=x^10-1;
  q:=x-1;
  divide(p,q,b);
```

$$p := x^{10} - 1$$
$$q := x - 1$$
$$true$$

We assign the polynomials $x^{10} - 1$ and $x - 1$ to the variables `p` and `q`, respectively, and then ask Maple to divide `p` by `q`.[5] Maple replies with the statement `true` to indicate that the polynomial $q = x - 1$ divides the polynomial $p = x^{10} - 1$ exactly, and places the quotient in the third argument of the `divide` command. Thus, if you want to know the value of the quotient, you must see what the variable `b` stands for now:

```
> b;
```

$$x^9 + x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$$

Of course, the answer produced by `divide` will be `false` if the polynomials do not divide each other exactly. Then one can use the commands `quo` and `rem` to find the *quotient* and *remainder* of the division:

```
> r:=x^2+2*x+3;
```

[4] In the worksheet interface, we can insert linebreaks by holding the *shift* key and the *return* key simultaneously. Maple does not execute what we type until we hit the return key unshifted.

[5] Of course, we could have done all of this in a single Maple command `divide(x^10-1, x-1, b);` if we had no intention of using `p` or `q` again.

$$r := x^2 + 2x + 3$$

```
> divide(p,r,c);
```

$$false$$

```
> quo(p,r,x);
```

$$x^8 - 2x^7 + x^6 + 4x^5 - 11x^4 + 10x^3 + 13x^2 - 56x + 73$$

```
> rem(p,r,x);
```

$$-220 + 22x$$

We would have obtained an error if we would have tried `divide(p,r,b)`, trying to place the quotient of the division in the variable `b`: it is no longer a variable since it has a value from the previous example:

```
> divide(p,r,b);

Error, wrong number (or type) of parameters in function divide
```

The above command is equivalent to `divide(p,r,x^9+x^8+x^7+x^6+x^5+x^4+x^3+x^2+x+1)`, which makes no sense at all. If we insist on calling the result of the division `b`, it would have been necessary to *unassign* the value of `b` first:

```
> b:='b';
```

$$b := b$$

We could also have done this within the command itself, using `divide(p,r,'b')`.[6] Further discussion on this is given in section 4.2.

We can use `solve` for systems of equations. For example:

```
> solve({1*x+(1/2)*y+(1/6)*z =1,
        (1/2)*x+(1/6)*y+(1/24)*z =0,
        (1/6)*x+(1/24)*y+(1/120)*z =1},  {x,y,z} );
```

[6]In both cases, it is very important to use the proper quotation marks; using " or ‘ means a very different thing, and would not work. The quotation mark used here is the ‘single quote’ or ‘right quote’, and on most keyboards is on the same key as the double quoute, near the enter key.

$$\{x = 39, y = -216, z = 420\}$$

Maple also solves inequalities:

```
> solve(x^2-4*x-7>0,x);
```

$$\text{RealRange}(-\infty, \text{Open}(2-\sqrt{11})), \text{RealRange}(\text{Open}(2+\sqrt{11}), \infty)$$

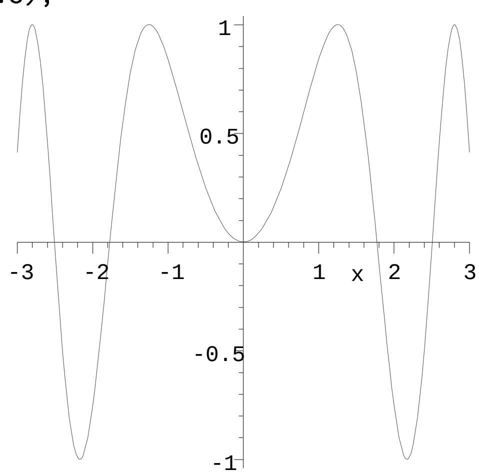
```
> solve(abs(1-x^2)<1/2,x);
```

$$\text{RealRange}\left(\text{Open}\left(-\frac{\sqrt{6}}{2}\right), \text{Open}\left(-\frac{\sqrt{2}}{2}\right)\right), \text{RealRange}\left(\text{Open}\left(\frac{\sqrt{2}}{2}\right), \text{Open}\left(\frac{\sqrt{6}}{2}\right)\right)$$

2.3 Graphing in Maple

Let us now use the graphics facilities of Maple. It is quite straightforward to draw the graph of a simple expression. For example, to graph the function $\sin(x^2)$ over the range $-3 \le x \le 3$, we do the following.

```
> plot(sin(x^2), x=-3..3);
```



Notice that within the worksheet interface, clicking the right mouse button on the displayed graph causes a menu to pop up that allows you to change some of its properties. Clicking the left mouse button causes a box to appear around the graph and the buttons on the toolbar to change. You can use the box to resize the graph, and the buttons to change attributes such as the style of the axes and the aspect ration.

In section 9 we shall discuss in further detail the graphing features of Maple.

3 The Maple worksheet

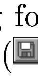
Maple's graphical interface allows you to keep a collection of commands and their results in one place, to save your work, and resume previous sessions. You can also add comments and text to your worksheet, to make it more readable and to describe the process of solving the problem. It also enables you to structure your worksheet into sections of related items.

3.1 Introducing the worksheet

As we have seen, you enter Maple commands at the `>` prompt, and the results appear in your worksheet. If you go back and change a Maple command and re-execute it, the result in the worksheet changes.

At the top of the screen is a menu bar (with entries like `File`, `Edit`, and so on): clicking on each of these words gives you a menu which allows you to affect your session in various ways. For example, clicking on the `File` menu and selecting `Save` will allow you to save your current worksheet for later use; selecting `Open` from the same menu allows you to load an existing worksheet.

Figure 2: The menu bar, button bar, and context bar.

Below the menu bar is a collection of buttons which are short-cuts for items found on the various menus. Clicking on one of these is much quicker than searching for the command on the proper menu. For example, the button that looks like a floppy disk () saves the current worksheet, and the magnifying glasses () adjust the zoom factor (or magnification) of the current worksheet. The stop sign interrupts a computation, which is very useful if you ask Maple to do something that takes a very long time, such as to find the factors of $2^{1000} - 1$.

Below this is the "context bar", which changes depending on what kind of item you currently have selected. For example, as mentioned in §2.3, when graphics are selected there are buttons to adjust the style of graphics (points or lines for 2-dimensional graphics, shading on 3-dimensional graphics), axes style, and so on. When a text item or comment section is chosen, selections for typeface, centering, and so on appear.

3.2 Worksheet basics: the front-end and the kernel

It is important to remember that the worksheet is only a record of a Maple session, not the session itself. Since you have the ability to edit and reorder it, it can sometimes appear to contain inconsistent information. Here is a small example:

First, suppose we issue the following commands:

```
> one:= sin(x)^2 + cos(x)^2;
```

$$one := \sin(x)^2 + \cos(x)^2$$

```
> diff(one,x);
```

$$0$$

The command `diff(one,x)` asks Maple to compute the derivative of the expression called `one` with respect to the variable `x`. The result is, of course, 0 because $\sin^2 x + \cos^2 x$ is a constant.

Now, we go back and change the `+` in the definition of `one` to a `-`, and execute the statement. However, we do not execute the `diff` command. Our worksheet now looks like this:

```
> one:= sin(x)^2 - cos(x)^2;
```

$$one := \sin(x)^2 - \cos(x)^2$$

```
> diff(one,x);
```

$$0$$

This seems to contain an error (the derivative should be $4\sin(x)\cos(x)$). Obviously, this is because we we didn't execute all the statements. If you place your cursor on the line with `diff(one,x)` and hit the return key, the worksheet will be correct again.

This may seem like a silly discussion, but this apparently obvious example is to make an important point: Maple only "sees" your commands in the order you execute them, *not* in the order you see them in the worksheet. Also, if you load a worksheet from a previous session (or made by someone else), any assignments and results will not be accessible to the Maple session until you execute those statements, *even though the results appear on your screen.*

To help you understand why this is, we point out that a Maple session typically consists of two processes (or programs) which communicate with each other: the Maple kernel and the "front-end", or user interface. The front-end is the part that interacts with the user, accepting your commands, showing the results of your computations, and so on. The kernel is the part that does the actual computations. In certain situations (for example, when using a "parallel Maple kernel"), it is possible to have more than one Maple kernel associated with the same front-end.

3.3 Online Maple help

Maple has an extensive and very useful online help system, containing information on all of the commands, as well as a number of tutorials and example worksheets.[7] Spending some time exploring the help system is well worth the effort. There are a few ways to get into the system; one obvious one is to select from the `Help` menu (on the right of the menu bar). But if you know the specific item you want help on, you can also do this in a quicker way.

Suppose you want information on Maple's `factor` command. Enter the Maple command

```
> ?factor
```

(here the semicolon is optional). Then Maple creates another window that very likely will contain more information than you really need or care about. You can read all or part of the information displayed in this window by moving the text down with the *scroll bar* to the right of the window. You can also click in the help window, and use the arrow keys on your keyboard.

You should notice that at the end of the help window you will find a good set of examples on the uses of the command you are inquiring about. This examples can be copied and pasted in your worksheet and executed. In this way, you can see what the command really does.

Usually there is also a section labeled "See Also", which points you to related commands. Each of these words is a link to another help page. It works like a web browser: clicking the underlined word takes you to the help page on that topic.[8] You can use the back button ( ) to return to the previous help page.

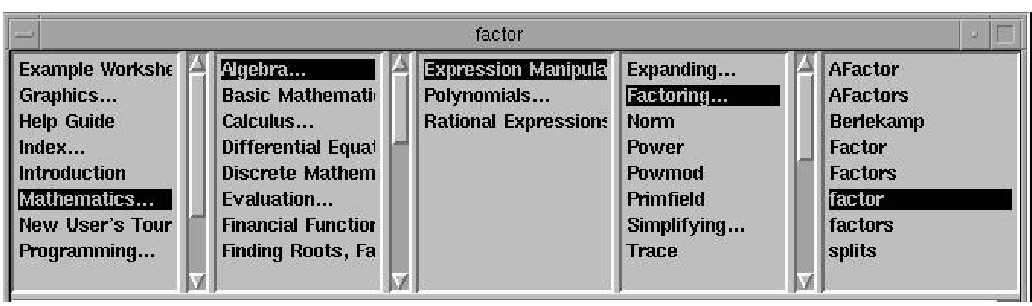


Figure 3: Maple's help index.

At the top of each help page is the help index (see Fig 3), which is a table of contents for the entire help system, organized by topic. You can use this to browse for commands you might not otherwise have known about.

Finally, on the `Help` menu are entries called "topic search" and "full text search". The first looks for help on a specific topic, and the latter searches the text of every page in the help system for the word you ask. Both of these can be valuable resources when used properly.

[7] There is even help on the help system. Select `Using Help` from the `Help` menu, or give the command `?helpguide`.

[8] If you prefer, holding the Control key while clicking on the link opens the page in a new help window, allowing you to view both at the same time.

3.4 Documenting and structuring your worksheet

Aside from the great advantage of making it easy to keep track of your commands and their results, the worksheet has some features of a word processor, allowing you to insert comments and text which describes your solution (or anything else you like). This can be very important, especially if you intend to show your worksheet to someone else (or even look at it yourself later).

To enter text into a worksheet, click on where you'd like it to go, and hit the text-mode button (it looks like T). Note that this button will convert a Maple input region (that is, one with a `>` prompt) to a text region— you'd probably prefer to *insert* your comments rather than change your commands to comments. To do this, either click before the `>` prompt, or select `Paragraph...Before` from the `Insert` menu.

In the text region, you can type whatever you like, using the context bar to adjust formatting such as the typeface, centering, etc. You can even include mathematical expressions in your text: click the "math input" button (Σ)[9] and type the Maple expression for the math you want. When you are done, use the text-mode button to return to entering text. You can also paste the output from a previously executed Maple command into the text area.

The worksheet interface also allows you to group parts of your worksheet into sections. These sections can contain paragraphs and Maple execution groups, and subsections. You can recognize a section by a large square bracket to the left, with a box at the top of it. Clicking on the box collapses the section, showing only the its title. Collapsing all sections allows you to see the structure of your document in outline form. To expand a collapsed section, click on the box next to the title.

For an extensive description of how to document and structure your worksheets, see the "Documenting your Work" section in the online help. This can be invoked with `?documenting`.

4 Assignments, Functions and Constants

Let us discuss the syntax of some basic Maple commands, as well as some functions and constants that Maple knows.

4.1 Assignment statements

The assignment is the most basic and used of Maple statements. It allows you to give names to expressions you want the computer to remember for later use. It has the *colon-equals* format `name := expression`. The thing on the left side of `:=` is a *name*, i.e. a sequence of letters and digits that begins with a letter. Certain names must be avoided because Maple uses them for special purposes (see section 4.4):

[9] Or select `Math Input` from the `Insert` menu.

```
> a:=3; b:=5; c:=22/7;
```

$$a := 3$$

$$b := 5$$

$$c := 22/7$$

Now you can use `a`, `b` and `c` as though they are numbers. Thus, we have

```
> a+b;
```

$$8$$

```
> c*a;
```

$$\frac{66}{7}$$

```
>  a-b^2;
```

$$-22$$

4.2 Variables and subsequent assignments: unassignments

If you type a variable name and a semicolon, Maple will tell you what the variable stands for:

```
> c;
```

$$\frac{22}{7}$$

```
> x;
```

$$x$$

If you specify `y` as an expression involving `x` and then assign a particular value to `x`, this value will get substituted into `y`:

```
> y:=sqrt(x+3);
```

$$y := \sqrt{x+3}$$

```
> x:=sin(w);
```

$$x := \sin(w)$$

```
> y;
```

$$\sqrt{\sin(w)+3}$$

If `x` gets changed, then `y` will also change. Maple remembers the original definition of `y`:

```
> x:=cos(u);
```

$$x := \cos u$$

```
> y;
```

$$\sqrt{\cos u+3}$$

Notice that Maple evaluates `x` before telling you what the variable `y` stands for now. To undo this kind of thing, you can *unassign* variables. To do this for `x` execute:

```
> x:='x';
  y;
```

$$x := x$$

$$\sqrt{x+3}$$

We can also tell Maple to forget entirely all assignments made so far using the `restart` command.

```
> restart;
  x; y; z;
```

$$x$$

$$y$$

$$z$$

Surrounding an expression in single quotes[10] tells Maple to delay evaluation of it. This is why `x:='x'` unassigns `x`— `x` is assigned to mean just `x` without any evaluation. Note that stopping evaluation does not stop simplification:

```
>  'x + 2 + 5';
```

$$x + 7$$

4.3 Functions known to Maple. How to define your own functions

Maple has all the standard mathematical functions in its library. You can make use of them by their appropriate name. We list some of these.

- Trig and inverse trig functions: `sin`, `cos`, `tan`, `cot`, `sec`, `csc`, `arcsin`, `arccos`, `arctan`, `arccot`, `arcsec`, `arccsc`.

- Exponential and logarithm: Maple uses `exp(x)` for e^x and either `log(x)` or `ln(x)` for the natural logarithm $\ln x$. Logarithm in other bases are also available. For example, `log[10](x)` is $\log_{10} x$.

- Hyperbolic functions: `sinh`, `cosh`, etc.

- Square roots and absolute values: `sqrt(x)` and `abs(x)`.

A much more complete list can be found by asking Maple for help on the initially known functions; `?inifcns` does this.

Of course, you will find yourself in a situation where you would like to define your own functions. Suppose you are analyzing the expression $\sqrt{1+x^2}$, plugging several values into it and calculating the result. Maple does not have a built-in function to do this, but you can define one yourself:

[10]Be careful! Most keyboards have three different kinds of quotation marks, and they each mean something different to Maple. The single quote (') delays evaluation, the double quote (") indicates a string, and the backquote (`) indicates a name (similar to a string).

```
> f:=x->sqrt(1+x^2);
```

$$f := x \to \sqrt{1+x^2}$$

Carefully notice the syntax. `f` is the name of the function and is written to the right side of `:=`. The function f sends the variable x to the expression $\sqrt{1+x^2}$; this is indicated by the arrow. The arrow `->` is made with the minus and greater-than keys.

After such an assignment, you can use the name `f` in the same way you use other names of functions known to Maple, e.g. `exp` or `sin`:

```
> f(0); f(1); f(-1); f(3*x+1);
```

$$1$$

$$\sqrt{2}$$

$$\sqrt{2}$$

$$\sqrt{2+9x^2+6x}$$

We are so accustomed to the notation $f(x)$ to refer to a function that we might accidentally use it in Maple. Never try to define a function with an assignment like

```
> f(x):=sqrt(x-2);
```

$$f := x \to \sqrt{x-2}$$

It seemed like this worked, but in fact, the resulting `f(x)` does not work as expected. You will not be able to substitute into it or do anything you normally do with functions.[11]

Maple uses parentheses `( )` to group expressions together in a convenient form as in

```
> 2*(x+4)^3;
```

$$2(x+4)^3$$

or to *delimit the arguments of a function.* In mathematics, we sometimes cheat and write $\sin x$ instead of $\sin(x)$. In Maple, this is not permissible: you must always write `sin(x)`. Nested parentheses are permitted:

[11] Maple uses this notation for a "remember table"— you can use it to set specific values for the function. For example, if we first define f by `f:=x->sin(x)/x`, then asking for Maple for `f(0)` will give an error, since it is not a permissible value for the function. However, we can define $f(0) = 1$ with the command `f(0):=1`; after this, `f(0)` will be `1`, while $f(x) = \sin(x)/x$ for all other values of x.

```
> f(x);
```

$$\sqrt{1+x^2}$$

```
>  sin(sqrt(x+2)+ln(abs(x)));
```

$$\sin(\sqrt{x+2}+\ln|x|)$$

4.4 Special constants and reserved names

Maple knows several constants. To find them out, execute the command

```
> constants;
```

$$false, \gamma, \infty, true, Catalan, FAIL, \pi$$

The names of these constants are `false`, `gamma`, `infinity`, `true`, `Catalan`, `FAIL` and `Pi`, respectively. Be careful! Maple is case sensitive. Therefore `pi` is not the same as `Pi`. For example,

```
> cos(Pi/4);
```

$$\frac{\sqrt{2}}{2}$$

```
> cos(pi/4);
```

$$\cos\left(\frac{1}{4}\pi\right)$$

Maple also uses `I` to represent the complex number $\sqrt{-1}$. Earlier versions used `E` for `exp(1)`; this is no longer the case. If you'd like to add `E` as a constant, you can do the following:

```
> E:=exp(1);
  protect(E);
  constants := constants, E;
```

$$E := e$$

$$constants := false, \gamma, \infty, true, Catalan, FAIL, \pi, e$$

While protecting your new constant `E` will stop you from accidentally redefining it, adding `E` to the list of known constants doesn't really affect things very much.

There are many commands and special objects in Maple that have preassigned names. In creating your own expressions and setting the name of your own variables, you cannot use any of the following 30 keywords:

```
and    by     do         done   elif    else   end    fi    for     from
if     in     intersect  local  minus   mod    not    od    option  options
or     proc   quit       read   save    stop   then   to    union   while
```

You can check this list for yourself using the help system by giving the command `?keywords`.

Maple also starts with a large number of predefined functions and constants. Many of these can be listed with `inifcns` and `ininames`. If you try to use one by mistake, you will receive an error:

```
> gamma:=x^2+5;
```

```
Error, attempting to assign to name `gamma` which is protected
```

If you insist, you can use `unprotect` to allow you to assign to this name. But **don't do this** unless you know what you are doing; this changes the meaning of this name even when it is used internally, and can have unexpected consequences.

5 The limit command

The limit command is used to compute limits. Its syntax is basically

```
> limit(f,x=a);
```

where `f` is a Maple algebraic expression which (in general) depends on the variable `x` and `a` is the expression which `x` approaches. For example, to compute

$$\lim_{x\to 2}\frac{3x-6}{x^2-4},$$

we execute the Maple command

```
> limit((3*x-6)/(x^2-4),x=2);
```

$$\frac{3}{4}$$

Maple can also deal with limits which do not exist, for example $\lim_{x\to 0}\frac{1}{x}$:

```
> limit(1/x,x=0);
```

$$undefined$$

In the following exercise, we define the slope of a straight line passing through the points (x_1, y_1), (x_2, y_2), use this function to find the slope $m(x)$ of the line passing through the points $(1,1)$ and (x, x^2), and finally compute the limit of $m(x)$ as $x \to 1$:

```
> slope:=(x1,y1,x2,y2)->(y2-y1)/(x2-x1);
  m:=x->slope(1,1,x,x^2);
```

$$slope := (x1, y1, x2, y2) \to \frac{y2 - y1}{x2 - x1}$$

$$m := x \to slope(1, 1, x, x^2)$$

```
> limit(m(x),x=1);
```

$$2$$

Maple is sometimes unable to determine a limit or whether it exists. In such a case, it will return nothing after you execute the limit command.

Maple computes *two-sided* limits. For example, if you specify the argument `x=1` as in the previous example, Maple assumes you mean that `x` approaches `1` from either the right or the left through real values only (as opposed to complex ones). However, Maple can compute one sided and complex limits also:

```
> limit(1/x,x=0,right);
```

$$\infty$$

```
> limit(1/x,x=0,left);
```

$$-\infty$$

```
> limit(x*log(x),x=0,complex);
```

$$0$$

You can also compute limits as $x \to \infty$ and limits of functions of more than one variable:

```
> limit(arctan(x),x=infinity);
```

$$\frac{\pi}{2}$$

```
> limit(x/(x^2+y^2),{x=0,y=0});
```

$$undefined$$

A common error is to try to compute the limit of a function `f(x)` when `x` has been previously given a value. If you find yourself in this situation, unassign the value of `x` executing the command `x:='x';`. The previous example will fail if `y` still has the value assigned to it in section 4.2.

6 The diff (and Diff) command

The `diff` command is used to compute derivatives of Maple expressions. Its syntax is basically

```
> diff(f,x);
```

where `f` is an algebraic expression and `x` is the variable with respect to which the derivative is taken. For example, to compute

$$\frac{d}{dx}\frac{3x-6}{x^2-4},$$

we execute the Maple command

```
> diff((3*x-6)/(x^2-4),x);
```

$$\frac{3}{x^2-4} - 2\frac{(3x-6)x}{(x^2-4)^2}$$

You must specify the variable with respect to which you wish to take the derivative because, almost all the time, there are constants and other parameters around:

```
> diff(exp(a*x),x);
```

$$ae^{ax}$$

Maple knows theoretical results about derivatives. For example, even if the functions `g(x)` and `h(x)` are not defined, Maple will give you the derivative of their product in terms of the functions and their derivatives:

```
> diff(h(x)*g(x),x);
```

$$\left(\frac{\partial}{\partial x}h(x)\right)g(x)+h(x)\left(\frac{\partial}{\partial x}g(x)\right)$$

6.1 Higher order derivatives

To compute higher order derivatives, you can of course iterate the above command:

```
> diff(diff(3*sin(x),x),x);
```

$$-3\sin(x)$$

Such a command first calculates the derivative with respect to `x` of the expression `3*sin(x)` and then differentiates the result to obtain the second derivative. The same result can be accomplished with either one of the following commands:

```
> diff(3*sin(x),x,x);
```

$$-3\sin(x)$$

```
> diff(3*sin(x),x$2);
```

$$-3\sin(x)$$

There is an obvious extension of this command when calculating more than two derivatives. You can also calculate partial derivatives of higher order:

```
> diff(ln(x)*exp(-2*x),x$3);
```

$$2\frac{e^{-2x}}{x^3}+6\frac{e^{-2x}}{x^2}+12\frac{e^{-2x}}{x}-8\ln(x)e^{-2x}$$

```
> diff(exp(x)*y^2*sin(z),x,y,z);
```

$$2e^x y\cos(z)$$

6.2 Implicit differentiation

Maple knows how to take derivative of both sides of an equation, and, as indicated above, Maple can also take derivatives "theoretically" even if the definition of the functions involved are not specified. However, you must always indicate *explicitly* the dependence of the functions on their variables. So, if you are thinking of `y` as a function of `x`, you must write `y(x)` in your equation rather than just `y`.

Here is a typical implicit differentiation problem. Consider the equation $x^2y - 3y^3x = 0$. Find the slope of the graph of the curve defined by this equation at the point $(3, 1)$:

```
> eq:=x^2*y(x)-3*y(x)^3*x = 0;
  deq:=diff(eq,x);
  solve(deq,diff(y(x),x));
```

$$eq := x^2y(x) - 3y(x)^3x = 0;$$

$$deq := 2xy(x) + x^2\left(\frac{\partial}{\partial x}y(x)\right) - 9y(x)^2\left(\frac{\partial}{\partial x}y(x)\right) - 3y(x)^3 = 0$$

$$-\frac{2xy(x) - 3y(x)^3}{x^2 - 9y(x)^2x}$$

```
> subs(y(x)=1,x=3,%);
```

$$\frac{1}{6}$$

The first command defines the equation relating `x` and `y`, the second defines an equation `deq` giving the relation between `x`, `y(x)` and the derivative of `y` with respect to `x`. Using `solve`, we solve `deq` to find the derivative as a function of `x` and `y(x)`, and in the result we substitute `x=3` and `y=1` to obtain the desired slope (see section 8 to learn about the `subs` command).[12]

Maple actually has a built-in command to do implicit differentiation, `implicitdiff`. Thus, we could have done this same problem more concisely using the single command

```
> subs({y=1,x=3},  implicitdiff(x^2y-3y^3x=0,y,x);
```

$$\frac{1}{6}$$

Occasionally, to make your worksheets easier to read, you may wish to have Maple display a derivative in standard mathematical notation without evaluating it. For this purpose there

[12] Again, the major caveat with the Maple derivative command is to avoid calculating derivatives with respect to a variable that has been previously given a value. In such an instance, unassign the value of the variable first.

is an *inert* capitalized form of the `diff` command: `Diff`. The two forms `diff` and `Diff` are usually combined to produce meaningful sentences:

```
> Diff(exp(x)/(1-x),x);
```

$$\frac{\partial}{\partial x}\frac{e^x}{1-x}$$

```
> Diff(exp(x)/(1-x),x)=diff(exp(x)/(1-x),x);
```

$$\frac{\partial}{\partial x}\frac{e^x}{1-x} = \frac{e^x}{1-x} + \frac{e^x}{(1-x)^2}$$

This command is particularly useful to produce easy to read worksheets that involve partial derivatives:

```
> f:=x^3*exp(y)-sin(x*y);
```

$$f := x^3 e^y - \sin(xy)$$

```
> Diff(f,x,x,y)=diff(f,x,x,y);
```

$$\frac{\partial^3}{\partial y \partial x^2} x^3 e^y - \sin(xy) = 6xe^y + \cos(xy)xy^2 + 2\sin(xy)y$$

7 The int (and Int) command

7.1 Symbolic integrals

The `int` command is used to compute both definite and indefinite integrals of Maple expressions. Its syntax is basically

```
> int(f,x);
```

where `f` is an algebraic expression and `x` is the integration variable. For example, to compute

$$\int \frac{3x-6}{x^2-4}\,dx\,,$$

we execute the Maple command

```
> int((3*x-6)/(x^2-4),x);
```

$$3\ln(x+2)$$

Notice that Maple does not provide the constant of integration. You will occasionally have to take this into account and provide your own constant.

You must specify the variable of integration. In expressions involving other parameters, Maple assumes that you want the integral of the expression as the variable you specify changes and that all other parameters in the expression represent constants:

```
> int(exp(a*x),x);
```

$$\frac{e^{ax}}{a}$$

To compute a definite integral, the *range* over which the integration variable moves must be specified:

```
> int(x^2*exp(x),x=0..2);
```

$$2e^2 - 2$$

The `int` command in Maple has a particular behavior in certain situations:

- If Maple cannot compute the integral in *closed form*, it will return it unevaluated:

  ```
  > int(ln(sin(sqrt(x^12-5*x^7+50*x+2))),x);
  ```

  $$\int \ln(\sin(\sqrt{x^{12} - 5x^7 + 50x + 2}))\, dx$$

- Sometimes the integral cannot be evaluated in closed form in terms of *elementary* functions, but the answer has a special name in mathematical circles due to its importance in applications. For example,

  ```
  > int(sin(2*x)/x,x);
  ```

  $$Si(2x)$$

  Here `Si` is the *special name* of one of these functions which appear frequently in mathematical physics. To learn more about it, ask Maple for help on it with the command `?Si`. This will bring up a window with information about the function `Si`. If Maple responds to an integral with one of these functions, it is quite likely that the integral cannot be evaluated in terms of elementary functions.

- Sometimes, a result may be expressed in terms of the roots of a polynomial which does not factor over the rationals. For example,

```
> int(1/(x^8 +1),x);
```

$$\sum_{\_R=\mathrm{RootOf}(1+16777216\_Z^8)} \_R\ln(x+8\_R)$$

In this answer, the sum is taken over all roots R of the polynomial $1+16777216z^8$, and the summand is $R\log(x+8R)$.

As with `diff`, there is an *inert* form `Int` of the integral command which can be used in combination with `int` to produce easily readable worksheets:

```
> Int(ln(1+3*x), x=1..4);
```

$$\int_1^4 \ln(1+3x)dx$$

```
> Int(ln(1+3*x), x=1..4)=int(ln(1+3*x),x=1..4);
```

$$\int_1^4 \ln(1+3x)dx = \frac{13}{3}\ln(13) - 3 - \frac{4}{3}\ln(4)$$

The inert form `Int` is also very useful in many situations when you wish to delay the evaluation of an integral, as we shall see below.

7.2 Numerical integration

With the `evalf` command, you can force Maple to apply a numerical approximation technique for definite integration:

```
> evalf(Int(sqrt(1+x^10),x=0..1));
```

$$1.040899075$$

Notice that the we are using the inert form of the integration command, `Int`. This prevents Maple from attempting to evaluate the integral symbolically and then applying `evalf` to the answer. In many cases, this can save a huge amount of time, because Maple will work very hard to try to compute the symbolic form of the integral. For example, approximating $\int_0^1 e^{\sin(x)}\,dx$ with the command

```
> evalf(int(exp(sin(x))),x=0..1);
```

took more than 10 times the amount of time needed to execute

```
> evalf(Int(exp(sin(x))),x=0..1);
```

More complicated integrals can have even more dramatic differences.

7.3 Approximations through Riemann Sums

Maple can also compute expressions that approximate a definite integral using rectangles, trapezoids, etc. It can also do approximations using Simpson's rule. The only difficulty is that these commands are contained in a separate library called `student` and you must load this library into computer memory before you can use it:

```
> with(student):
```

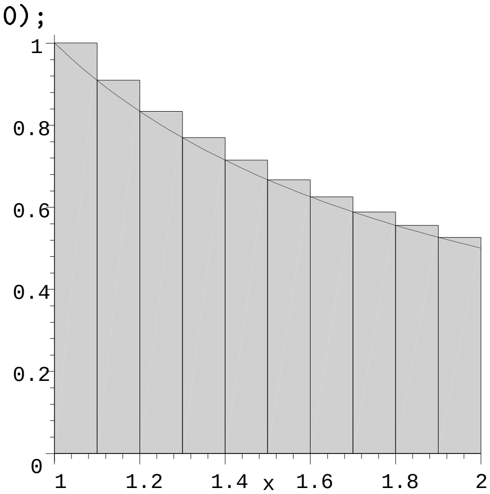
Once loaded, you can play with it. For example, if you want to use the left sum approximation to an integral, the height of each rectangle is determined by the value of the function at the left side of each interval. You may specify the number of intervals you wish to use. If you do not, Maple will use four intervals by default:

```
> leftsum(x^4*ln(x),x=1..4,10);
```

$$\frac{3}{10}\left(\sum_{i=0}^{9}(1+\frac{3}{10}i)^4\ln(1+\frac{3}{10}i)\right)$$

Maple can also draw pictures of the rectangles used to approximate the integral of a function on a given interval:

```
> leftbox(1/x,x=1..2,10);
```



The following is the result of using the Simpson's rule for the function $xsin(x^2)$ on the interval $1 \leq x \leq 5$. Since the number of intervals is not specified, Maple assumes four equal intervals by default:

```
> simpson(x*sin(x^2),x=1..5);
```

$$\frac{1}{3}\sin(1) + \frac{5}{3}\sin(25) + \frac{4}{3}\left(\sum_{i=1}^{2} 2i\sin(4i^2)\right) + \frac{2}{3}\left(\sum_{i=1}^{1}(1+2i)\sin((1+2i^2)^2)\right)$$

7.4 Multiple integrals

Maple does iterated integrals:

```
> int(int(x^2*y^3,x=0..y),y=2..3);
```

$$\frac{2059}{21}$$

This is literally an iterated integral: the integral of x^2y^3 with respect to x on the interval $[0, y]$ is nested inside the integral with respect to y on the interval $[2, 3]$, in effect calculating

$$\int_2^3 \int_0^y x^2y^3dxdy\,.$$

8 The subs command

You have learned in the previous sections that if you assign a particular value to a variable, Maple will remember it for the rest of the session until you unassign it. At times, you might like to *plug in* some values into an expression without altering the variable (or variables) involved. Maple has the `subs` command which reports the result of making such a substitution:

```
> y:=x+3;
```

$$y := x + 3$$

```
> subs(x=2,y);
```

$$5$$

Notice that Maple reports the value obtained when plugging in `x=2` into `y` without giving that value to `x` or altering `y` in any way:

```
> x;
```

$$x$$

```
> y;
```

$$x+3$$

You can make more than one substitution:

```
> z:=x+sin(2*h):
  subs(x=3*u+1,h=4*Pi*q,z);
```

$$3u+1+\sin(8\pi q)$$

In this example, the first assignment is ended with a colon : to suppress the display. You can do this whenever you want to make an assignment which you do not want to see displayed. Then, plugging $x = 3u+1$ and $h = 4\pi q$ into the expression for z, we obtain $3u+1+\sin(8\pi q)$. Notice that the values of **x** and **h** that we plug in do not need to be enclosed in braces when issuing the command.

You should be aware of a few things when using the **subs** command:

- You need not fear *circular substitutions*:

  ```
  > z;
  ```

  $$x+\sin(2h)$$

  ```
  > subs(x=2*x,z);
  ```

  $$2x+\sin(2h)$$

  On the other hand, long chains of self-referential substitutions may produce results which are hard to predict:

  ```
  > subs(h=x,x=z,z);
  ```

  $$x+\sin(2h)+\sin(2x+2\sin(2h))$$

  Simultaneous substitutions are made enclosing the values you plug in in braces. This is in contrast with the left-to-right substitution above:

```
> subs({h=x,x=z},z);
```

$$x + \sin(2h) + \sin(2x)$$

- In certain situations, `subs` will not be able to find the expression you are substituting for if it is complicated. In those cases, try to *reword* your substitution request so that Maple *understands it*, or break the task into several manageable ones. In other cases, you might consider using `algsubs`, which allows you to substitute one algebraic expression for another.

- `subs` does what is called "syntactic substitution"; it substitutes one set of symbols for another. If you are using it to evaluate an expression with certain values "plugged in", using `eval` might be more appropriate. Here is an example that should illustrate the distinction:

```
> integral := int(f(t,a), t=a..x);
```

$$\int_a^x f(t,a)\,dt$$

```
> eval(integral,{t=0,a=1});
```

$$\int_1^x f(t,1)\,dt$$

```
> subs({t=0,a=1},integral);
```

$$\int f(0,1)\,d0 = 1..x$$

The seemingly nonsensical answer using `subs` above is because Maple interpreted the expression as meaning `int(f(0,1),0=1..x)` — what you get by replacing `t` by `0` and `a` by `1` without trying to interpret the expression.

8.1 Why is subs useful?

Often it is better to use `subs` rather than to make assignments to the variables in an expression. For example, if

```
> y:=3*x^2+5*x+2;
```

$$y := 3x^2 + 5x + 2$$

and you want to compute the difference quotient $(y(x+h) - y(x))/h$, then you should execute

```
> dq:=(subs(x=x+h,y)-y)/h;
```

$$dq := \frac{3(x+h)^2 + 5h - 3x^2}{h}$$

because if we set

```
> x:=x+h;

Warning: Recursive definition of name
```

$$x := x + h$$

any call to `y` will result in

```
> y;

Error, too many levels of recursion
```

In earlier versions of Maple, this action would have crashed your session.

On the other hand,

```
> x:=4;
```

$$x := 4$$

```
> y;
```

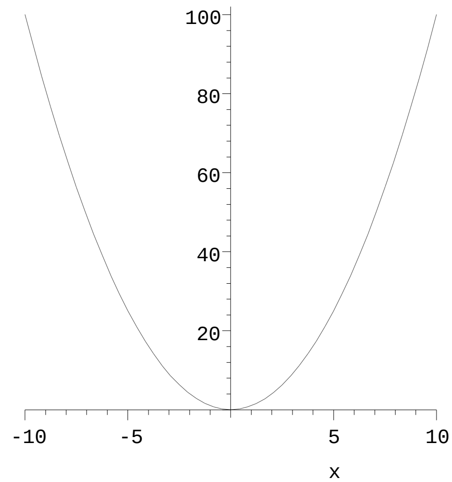
$$70$$

is alright, but now `y` is a number and we cannot plug any other values of `x` into it. Using `subs` is a better way to do the plugging in, because we need not unassign `y`.

9 Plotting with Maple

Let us now use the graphics facilities of Maple:

```
> plot(x^2, x=-10..10);
```



Maple displays the graph of the function x^2 from $x = -10$ to $x = 10$ as requested. In the worksheet interface, clicking the right mouse button on the graph pops up a menu with various options which allow you to manipulate your graph, and/or change its appearance in a suitable way. Clicking the left button "selects" the graphic, drawing a black box with "handles" around it. You can drag the mouse button on these to resize the plot. Also, the coordinates of the point where you clicked appear in the upper left of the worksheet. Selecting a graphic also changes the context bar: shortcuts for manipulating the axes, aspect ratio, and plotting style appear as buttons along the top.

The basic syntax of the plot command is

```
> plot(f,range);
```

where `f` is the expression to be plotted and `range` is the range of the parameter(s) for which you would like to see the plot of `f`. In the example above, we indicated the range of `x` by `x=-10..10`. Maple automatically chooses a scale on the vertical axis. There are many options to the `plot` command; while we will mention a few of them, the on-line help system has all the details.

Try both of the two similar plot commands below:

```
> plot(x^2-x, x=-1..2, y=-1..2);
  plot(x^2-x, x=-1..2,   -1..2);
```

In both cases, the plot is displayed for values of `y` between -1 and 2, but in the first, the variable `y` is explicitly named in the command, and the vertical axis gets a label. Whenever you are plotting an expression, the variable in the domain of the function *must* be specified.

9.1 Plotting several functions (or curves) on the same axes

We can plot more than one function (or curve) on the same coordinate system. Consider the following example, recalling the slope function of the line passing through the points (x_1, y_1), (x_2, y_2), used to define the slope $m(x)$ of the line passing through $(1, 1)$ and (x, x^2):

```
> slope:=(x1,y1,x2,y2)->(y2-y1)/(x2-x1);
  m:=x->slope(1,1,x,x^2);
```

$$slope := (x1, y1, x2, y2) \to \frac{y2 - y1}{x2 - x1}$$

$$m := x \to slope(1, 1, x, x^2)$$

We now define the equation of the lines passing through $(1, 1)$ and (x, x^2) for $x = 2, 3, 4, 5$ and plot the four lines together with the function `x^2`.

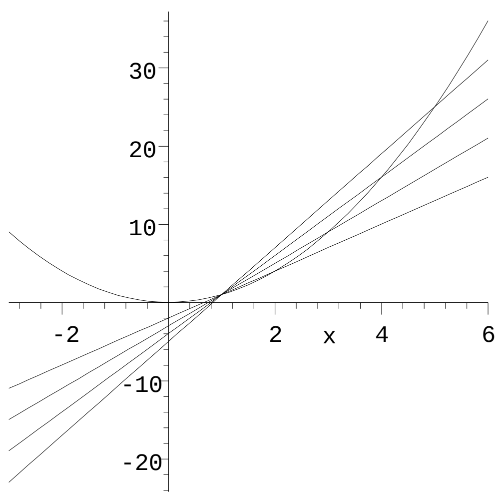
```
> line1:=m(2)*(x-1)+1;
  line2:=m(3)*(x-1)+1;
  line3:=m(4)*(x-1)+1;
  line4:=m(5)*(x-1)+1;
  plot({x^2,line1,line2,line3,line4},x=-3..6);
```

$$line1 := 3x - 2$$

$$line2 := 4x - 3$$

$$line3 := 5x - 4$$

$$line4 := 6x - 5$$



Observe that the expressions to be plotted are *enclosed* in braces `{ }`. Besides specifying the range on which `x` varies as done above, you can also specify and label the `y` range proceeding exactly in the same manner as when plotting a single function.[13]

[13]We could have done this all in a single plot command, either explicitly typing each of the lines, or using `seq` to generate them: `plot({x^2, seq( m(i)*(x-1)+1, i=2..5)}, x=-3..6);`

9.2 Fancier plotting

The `plot` command is very powerful, and you should look for details about it using the online help facility; the command `?plot` will bring this up. There are also several related pages, and some instructive example worksheets that are well worth looking at.

To make you aware of some of its features, we will discuss a few of them here:

- It is possible to have Maple *plot points*. This can be done in two ways depending upon how the points are generated. If you have a list of specific points, you can assign them to a name and then plot them as follows:

```
> points:=[[1,2],[1.5,1],[2,-1],[2.5,0.5],[3,1],[3.5,0.6],[4,0.2]]:
  plot(points,style=POINT);
```

  If `style=POINT` is replaced by `style=LINE`, the dots will be connected by segments of lines. In the worksheet interface, you can switch from one style to the other by first clicking the left mouse button on the graph, and then hitting the relevant button on the buttonbar.

  On the other hand, if the points result from evaluating an expression at several values of `x`, you can use `plot` in its usual form, but specifying `style=POINT`:

```
> plot(x^2,x=-2..2,style=POINT);
```

  Approximately 50 points will be plotted this way. You may notice that they will not usually be equally spaced, but concentrated in areas where the graph curves more. You can insist that more points be plotted by using the `numpoints` option:

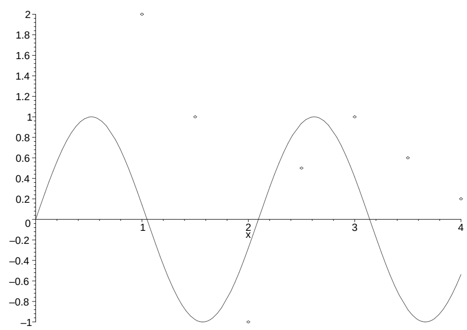
```
> plot(x^2,x=-2..2,style=POINT,numpoints=150);
```

- Several more specialized plotting functions are contained in a separate library called `plots`. For basic plotting, there are two commands from this library that are especially useful. They are `textplot` and `display`. To load these commands into the computer memory, execute the statement

```
> with(plots,textplot,display);
```

  The `display` command is useful to combine different kinds of plots into one picture. You can merge standard plots of expressions, plots of points, plots of text (This is the utility of `textplot`: to label things), and animations. In the next example, we combine the point plot of the variable `points` defined above with a plot of `sin(3*x)`. We first define them as different plots and then display them together. Note that except for the third command, we end with a colon (:) in order to suppress the Maple output.

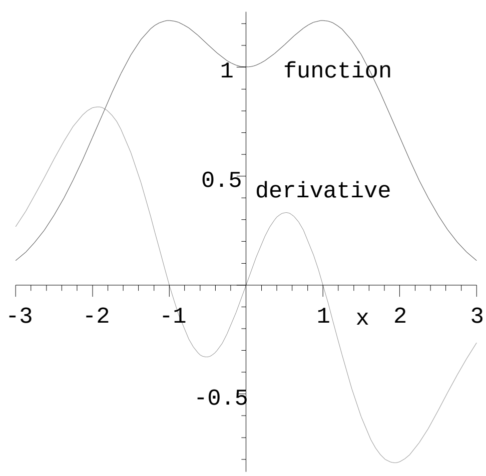
```
> plot1:=plot([points],style=POINT):
  plot2:=plot(sin(3*x),x=0..4):
  display({plot1,plot2},view=[0..4,-1..2]);
```



The `view` option controls the horizontal and vertical ranges to be displayed.

You can attach labels to your plots by combining `textplot` with `display`. The basic syntax of `textplot` is to use it with an argument of the form `[a,b,'name']` which places the word name inside the quotes on the plot so that it is centered at the point (a, b). For example:

```
> y:=(1+x^2)*exp(-x^2/2): d:=diff(y,x):
  F:=plot({y,d},x=-3..3):
  G:=textplot({[1,1,'function'],[0.75,0.45,'derivative']}):
  display({F,G});
```



We end with a word about plotting functions (as opposed to expressions), and a situation in which it is a good idea to do so. Sometimes you will have a relationship that you want to plot in the form of a function rather than an expression:

```
> f:=x->x^3*exp(-x):
```

In this situation, you can plot `f(x)` as explained above. Alternatively, you may use:

```
> plot(f, 0..3);
```

The two ways of plotting a function should not be confused. Neither of the following two statements will work correctly:

```
> plot(f(x), 0..3);

Plotting error, empty plot

> plot(f, x=0..3);
```

As you see, the first gives an error; the second gives a plot with nothing plotted on it.

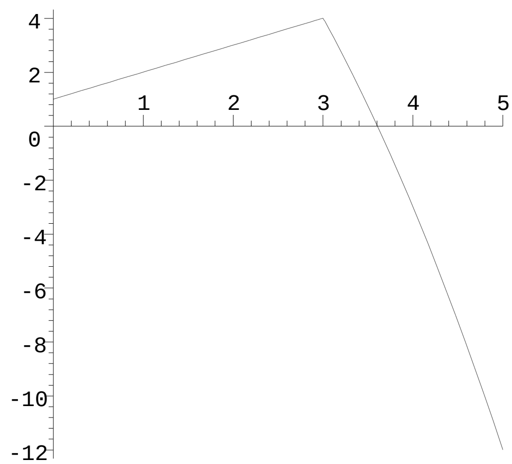
If you have a function defined using an *if-then* clause, you must use the function plotting command:

```
> f:=x-> if x<3 then x+1 else -x^2+13 fi:
  plot(f(x),x=0..5);

Error, (in f) cannot evaluate boolean
```

The error resulted because Maple attempts to understand the expression `f(x)` before it has a value for `x`. One way around this[14] is to use the command

```
> plot(f, 0..5);
```



You can see from the graph that the function `f` is probably continuous but not differentiable at `x=3`.

10 Exercises

1. Evaluate $\pi^{\sqrt{2}}$ to 30 decimal places.

2. Input the following as a Maple function:

$$f(x) = \frac{\sqrt{x}(-116x^3 + 8x^4 + 558x^2 - 891x)}{2x^5 - 29x^4 + 140x^3 - 225x^2}.$$

[14]We could also delay evaluation of `f(x)` using quotes, with `plot('f(x)', x=0..5);`. Alternatively, we could define `f` using `piecewise`, as `f:=x-> piecewise(x<3, x+1, -x^2+13);`.

a) Write this in a simpler form (that is, factor and reduce it).

b) Draw the graph of the function for x between 0 and 1). Adjust the vertical range so that some detail can be seen.

c) Compute the area of the part of the curve that lies above the x-axis and between $x = 0$ and $x = 5$ (that is, integrate the function over the appropriate range of x values). Give your answer both in actual form and as a decimal approximation to about 10 places.

d) What is the value of the integral if you remove the factor of $\sqrt{x}$ from the numerator?

e) Use the derivative of $f(x)$ to determine for what real value x the function has a local maximum. Use an approximation to about 8 decimal places.

3. Use the commands `seq` and `ithprime` to generate a list of the first 20 primes. Compute the sum of these 20 primes, and give its integer factors.

4. Find the solutions of the system of equations $\{x^2 - y^2 = 4, x - 2y = 2\}$.

5. Draw a graph showing both $\cos x$ and its fifth Taylor polynomial (that is, $1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4$) for x between -4 and 4. How many terms do you seem to need to get *good agreement* in this range.

   Hint: use a variation of the command `convert(taylor(cos(x),x,5),polynom)` to make this work.

   Think of a suitable way to demonstrate that the approximation you have taken is *good*.