MAT-544 Schul | Midterm | October , 2015

First Name: ____________________

Last Name: ____________________

Stony Brook ID: ____________________

Signature: ____________________

Write coherent mathematical statements and show your work on all problems. If you use a theorem from the book, you must fully state it. If you give an example/construction then you must prove it is such. Please write clearly.

Rules.

1. Start when told to; stop when told to.
2. No notes, books, etc,...
3. Turn OFF all unauthorized electronic devices (for example, your cell phone).

1 (20pts)	2 (10pts)	3 (10pts)	4 (10pts)	5 (15pts)

Choose 4 out of the 5 questions. Note that they are not all worth the same amount of points.

1. (20 points)
   (a) (5 points) State Egoroff's theorem.
   (b) (5 points) State Lusin's theorem.
   (c) (10 points) Use Egoroff's theorem to derive Lusin's theorem. Note: Upon request, I can give you the answer to (a)+(b) and deduct 10 points from your total.

2. (10 points) Let ν, μ be two positive, finite measures on the measure space $(\mathbb{X}, \mathcal{M})$. Let
$$\mathcal{F} = \{f \in L^+ : \int_E f d\mu \leq \nu(E), \quad \text{for all } E \in \mathcal{M}\}$$
   Show that there is a maximal function in $\mathcal{F}$.

3. (10 points) Let ν, μ be two positive, finite measures on the measure space $(\mathbb{X}, \mathcal{M})$. Suppose that $\nu \ll \mu$. Show that if $g \in L^1(\nu)$ then $g\frac{d\nu}{d\mu} \in L^1(\mu)$ and
$$\int g d\nu = \int g \frac{d\nu}{d\mu} d\mu \,.$$

4. (10 points)For $\mathcal{A} \subset 2^{\mathbb{X}}$, define the *monotone class* $\mathcal{C}(\mathcal{A}) \subset 2^{\mathbb{X}}$ as the minimal collection of sets closed under countable increasing unions and countable decreasing intersections.
   (a) State (do not prove) the *monotone class lemma*, a lemma relating $\mathcal{C}(\mathcal{A})$ and $\mathcal{M}(\mathcal{A})$ for an algebra $\mathcal{A}$.
   (b) Use the lemma from part (a) to show the following: Suppose $(\mathbb{X}, \mathcal{M}, \mu)$ and $(\mathbb{Y}, \mathcal{N}, \nu)$ are finite measure spaces. If $E \in \mathcal{M} \otimes \mathcal{N}$, then the function
$$x \to \nu(\{y : (x, y) \in E\})$$
   is measurable and
$$\mu \times \nu(E) = \int \nu(\{y : (x, y) \in E\}) d\mu(x)$$

5. (15 points)
   (a) (5 points) State the Hahn and Jordan decomposition theorems.
   (b) (10 points) Let ν, μ be two positive, finite measures on the measure space $(\mathbb{X}, \mathcal{M})$. Suppose $\nu \ll \mu$. Let $\epsilon > 0$ be given. Use the Hahn or Jordan decomposition theorems to explicitly give a **simple** function $f \in L^+$ such that for any $E \in \mathcal{M}$
$$\left| \int_E f d\mu - \nu(E) \right| < \epsilon$$

Note: do not use Lebesgue-Radon-Nikodym