MAT-550 Schul Midterm March , 2016

First Name: _____

Last Name: _____

Stony Brook ID: _____

Signature: _____

Write coherent mathematical statements and show your work on all problems. If you use a theorem from the book, you must fully state it. If you give an example/construction then you must prove it is such. Please write clearly.

Rules.

1. Start when told to; stop when told to.
2. No notes, books, etc,...
3. Turn OFF all unauthorized electronic devices (for example, your cell phone).

1 (30pts)		2 (10pts)	3 (10pts)	4 (10pts)	5 (10pts)

DO quesion number 1 (FIVE of the SIX parts).
Choose 3 out of the 4 questions numbered 2-5.

1. (**Do FIVE of the SIX parts of QUESTION 1.** Each part is worth 6 points)

   (A.) Suppose that $f \in C[0,1]$ is a real valued continuous function on $[0,1]$, such that

   $$\int_0^1 x^n f(x) dx = 0$$

   for all nonnegative integers n. Show that $f(x) = 0$ for all $x \in [0,1]$?

   (B.) State the Open Mapping Theorem

   (C.) State the Closed Graph Theorem

   (D.) Define Weak L^p

   (E) Give an example of a function f which is Weak L^3, but $f \notin L^3$.

   (F) State one of the two interpolation theorems we learned in class

2. (10 points)
Suppose X is a Banach space. Show that if X^* is separable, then X is separable.

3. (10 points) (You may use the open mapping theorem below.)
(a) Prove the Closed Graph Theorem

(b) Let $\ell^p(\mathbb{N},\mathbb{R}) := \{(x_1, x_2, ...) : \ x_i \in \mathbb{R}, \ \sum x_i^p < \infty\}$ and for $x \in \ell^p(\mathbb{N},\mathbb{R})$, set $\|x\|_p = (\sum |x_i|^p)^{\frac{1}{p}}$. Note that for $x \in \ell^1(\mathbb{N},\mathbb{R})$ we have $\|x\|_7 \leq \|x\|_1$. Is it true that there is a $C > 0$ such that $\|x\|_1 \leq C\|x\|_7$ for all $x \in \ell^1(\mathbb{N},\mathbb{R})$?

4. (10 points)
Let $F : \mathbb{R}^2 \to \mathbb{R}^2$ be a Lipschitz vector field. Suppose also that the maximal integral curves of this (time-independent) vector field are defined on the entire real line, i.e. $\Phi_F^t : \mathbb{R}^2 \to \mathbb{R}^2$ is defined for all $t \in \mathbb{R}$. Show that there is a constant $K > 0$ such that

$$\|\Phi_F^t(x) - \Phi_F^t(y)\| \leq e^{Kt}\|x - y\|\,.$$

5. (10 points)
Let $\epsilon > 0$ be given. Let $f : [0, 2] \to \mathbb{R}$ be continuous. Show that there is a polynomial p such that $p(k) = f(k)$ for $k \in \{0, 1, 2\}$ and for all $x \in [0, 2]$ we have $|f(x) - p(x)| < \epsilon$.