SOL'NS, SPR17

Directions: Show all work in the space provided. If you need more space use the blank backs of the exam sheets. Be sure that you don't have answers to any question in more than one place. Answers without the required work will receive no credit.

Part I: For the following find the indefinite or definite integrals. Leave all answers in simplest form. [10 points each]

Reference Formulas: You may not need all of these.

$$cos^2x = \frac{1}{2}(1 + cos2x) \qquad sin^2x = \frac{1}{2}(1 - cos2x)$$

$$sin^2x + cos^2x = 1 \qquad tan^2x = sec^2x - 1$$

$$a^2\,cos^2\theta = a^2 - a^2sin^2\theta$$

$$a^2\,tan^2\theta = a^2sec^2\theta - a^2 \qquad a^2\,sec^2\theta = a^2tan^2\theta + a^2$$

1. $\int_1^2 \frac{-2}{y^3}dy$ $= \int_1^2 -2y^{-3}dy = \frac{\cancel{-2}\,y^{-2}}{\cancel{-2}}\Big|_1^2$

$= \frac{1}{4} - 1 = \boxed{-3/4}$

2. $\int x\,sin(x^2+1)dx$

LET $u = x^2+1$

$du = 2x\,dx, \quad \frac{1}{2}du = x\,dx$

$= \frac{1}{2}\int \sin u\, du$

$= \frac{-1}{2}\cos(u) + C$

$= \boxed{-\frac{1}{2}\cos(x^2+1) + C}$

SOL'NS
SPR 17

3. $\int cos^2 x\, dx$ $= \int \frac{1}{2}(1+\cos 2x)\,dx$

$$= \boxed{\frac{1}{2}x + \frac{1}{4}\sin(2x) + C}$$

$$= \frac{1}{2}\left(x + \cos(x)\sin(x)\right) + C$$ IF YOU PREFER.

4. $\int \sin^2 x\, cosx\, dx$

LET $w = \sin x$
$dw = \cos x\, dx$

$$= \int w^2\, dw = \frac{1}{3}w^3 + C$$
$$= \frac{1}{3}\sin^3 x + C$$

5. $\int \frac{2}{x^2-2x}dx$ $= \int \frac{2}{x(x-2)}dx$

USE PARTIAL FRACTIONS:

$$\frac{2}{x(x-2)} = \frac{A}{x} + \frac{B}{x-2}$$

SO $2 = A(x-2) + Bx$

IF $x=0$, $2 = -2A + 0$, SO $A = -1$

IF $x=2$, $2 = 0 + 2B$, SO $B = 1$

CHECK: $\frac{-1}{x} + \frac{1}{x-2} = \frac{2-x+x}{x(x-2)}$ YEP!

$$\int \frac{2\,dx}{x(x-2)} = \int \frac{-1}{x}dx + \int \frac{1}{x-2}dx$$
$$= \boxed{-\ln|x| + \ln|x-2| + C}$$
$$= \ln\left|\frac{x-2}{x}\right| + C$$ IF YOU PREFER.

6. $\int \frac{2x+2}{x^2+2x-3}\,dx$ YOU COULD USE PARTIAL FRACTIONS

TO WRITE $\frac{2x+2}{x^2+2x-3} = \frac{2x+2}{(x-1)(x+3)} = \frac{1}{x-1} + \frac{1}{x-3}$,

BUT THAT'S TOO MUCH WORK.

INSTEAD, LET $u = x^2+2x-3$, ~~TO G~~

so $du = (2x+2)\,dx$.

$$\int \frac{du}{u} = \ln|u| = \boxed{\ln|x^2+2x-3| + C}$$

7. $\int x^2 \sin x\, dx$

HERE WE INTEGRATE BY PARTS, TWICE:

$$\boxed{\begin{array}{ll} u = x^2 & dv = \sin x\,dx \\ du = 2x\,dx & v = -\cos x \end{array}}$$

$= -x^2\cos x + \int 2x\cos x\,dx$

$u = x \quad dv = \cos x\,dx$

$du = dx \quad v = \sin x$

$= -x^2\cos x + 2\left(x\sin x - \int \sin x\,dx\right)$

$= \boxed{-x^2\cos x + 2x\sin x + 2\cos x + C}$

8. Evaluate the improper integral: $\int_0^{+\infty} e^{-x}dx$ If the integral diverges explain why, otherwise give the value of the integral.

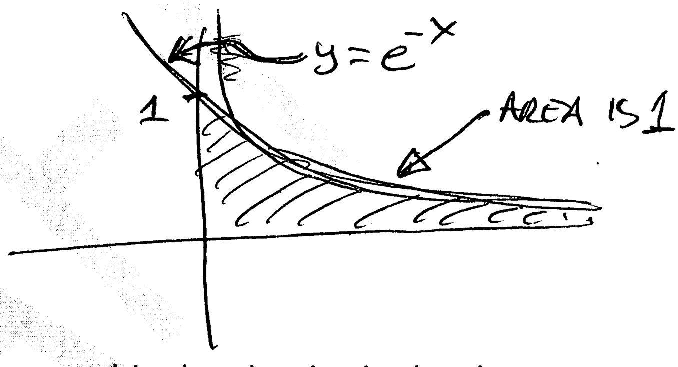
$$\int_0^{\infty} e^{-x}dx = \lim_{m\to+\infty} \int_0^m e^{-x}dx$$

$$= \lim_{m\to+\infty} \left(-e^{-x}\Big|_0^m\right)$$

$$= \lim_{m\to+\infty} -e^{-m} + 1 \quad (\text{SINCE } e^0 = 1)$$

$$= 0 + 1$$

$$= \boxed{1}$$



9. Evaluate the improper integral below. If the integral diverges explain why, otherwise give the value of the integral.

$\int_2^5 \frac{3dx}{x-2}$ IMPROPER SINCE $x-2=0$ WHEN $x=2$.

$$= \lim_{a\to2^+} \int_a^5 \frac{3}{x-2}dx$$

$$= \lim_{a\to2^+} \left(3\ln(x-2)\Big|_a^5\right)$$

$$= 3\ln 3 - 3\lim_{a\to2^+} \ln(a-2)$$

$$= 3\ln 3 + \infty$$

SINCE $\left(\lim_{x\to0^+} \ln(0) = -\infty\right)$

SO $\boxed{\text{INTEGRAL DIVERGES}}$

Sol'ns, SPR '17

10. Evaluate the integral $\int \frac{\sqrt{x^2-16}}{x}$ dx using a *trigonometric substitution*. See the page 2 of this exam for needed formulas.

LET $x = 4\sec\theta$

so $dx = 4\sec\theta\tan\theta\, d\theta$

so

$$\int \frac{\sqrt{x^2-16}}{x}dx = \int \frac{\sqrt{16\sec^2\theta - 16}}{4\sec\theta} \cdot 4\sec\theta\tan\theta\, d\theta$$

$$= \frac{\cancel{4}}{\cancel{4}}\int \frac{4\sqrt{\sec^2\theta - 1}}{\cancel{\sec\theta}} \cdot \cancel{\sec\theta}\tan\theta\, d\theta$$

$$= 4\int \tan^2\theta\, d\theta = 4\int(\sec^2\theta - 1)\, d\theta$$

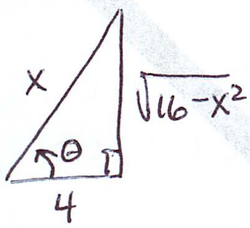
$$= 4\tan\theta - 4\theta + C$$

SINCE $x = 4\sec\theta$, $\theta = \operatorname{arcsec}(x/4) = \arccos(4/x)$

$= \arctan\left(\frac{\sqrt{16-x^2}}{4}\right)$

TO SIMPLIFY $\tan\theta$,

OBSERVE SINCE $\sec\theta = x/4$, $\tan\theta = \frac{1}{4}\sqrt{x^2-16}$



SO WE HAVE

$$4\left(\frac{1}{4}\sqrt{x^2-16}\right) - 4\operatorname{arcsec}(4/x) + C$$

ie

$$\boxed{\int \frac{\sqrt{x^2-16}}{x}dx = \sqrt{x^2-16} - 4\operatorname{arcsec}(4/x) + C}$$