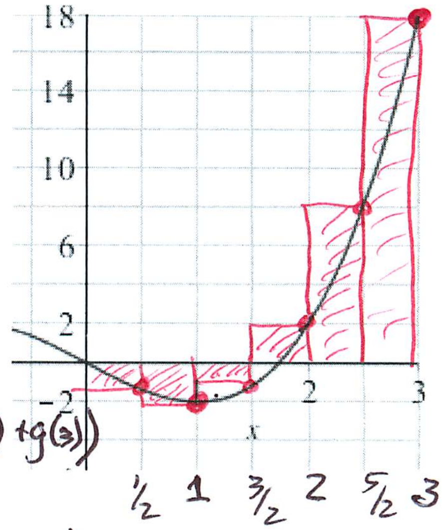
Soln's, SPR 15

Directions: Show all work in the space provided. If you need more space use the blank backs of the exam sheets. Be sure that you don't have answers to any question in more than one place. Answers without the required work will receive no credit. Simplify your answers.

1. For the function, g(x), whose graph appears below:

a) Estimate $\int_0^3 g(x)dx$ using a Riemann sum of 6 subintervals. Take the sample point to be the **right** endpoints in each interval. <u>Draw</u> the approximating rectangles and round approximate results correct to the *nearest integer*. [5]



$$\int_0^3 g(x)dx \approx \frac{1}{2}\sum_{i=1}^{n} g(x_i)$$

$$= \frac{1}{2}\left(g(\tfrac{1}{2}) + g(1) + g(\tfrac{3}{2}) + g(2) + g(\tfrac{5}{2}) + g(3)\right)$$

$$= \frac{1}{2}\left((-1) + (-2) + (-1) + 2 + 8 + 18\right)$$

$$= \frac{1}{2}(24) = \boxed{12}$$

b) The equation of the function above is $g(x) = x^3 - 3x$. Now find the exact value of $\int_0^3 g(x)dx$ by evaluating the integral *algebraically*. Remember the answer to a) is an approximation and the answer to b) is exact. [5]

$$\int_0^3 (x^3 - 3x)dx = \frac{1}{4}x^4 - \frac{3x^2}{2}\Big|_0^3$$

$$= \frac{3^4}{4} - \frac{3 \cdot 9}{2} = \frac{81}{4} - \frac{27}{2} = \boxed{\frac{27}{2}}$$

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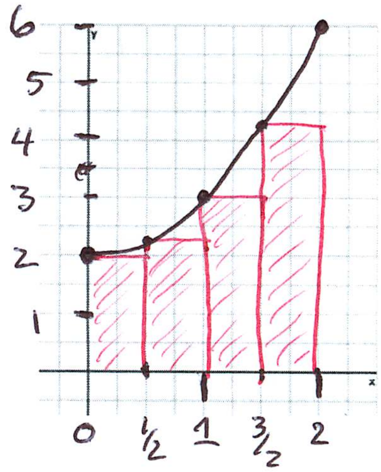
2. For the function f(x) = $x^2 + 2$

a) Sketch the graph from x = 0 to x = 2 on the grid provided. Pick an appropriate scale. [4]



b) Estimate the area under the graph of f from x = 0 to x = 2 using four rectangles where the height of each rectangle is the value of f at the **left-hand** side of its base. [5]

WE HAVE $\int_0^2 (x^2+2)dx \approx \frac{1}{2}\sum_{i=0}^{3} f(x_i) = \frac{1}{2}\left(f(0)+f(\tfrac{1}{2})+f(1)+f(\tfrac{3}{2})\right)$

$= \frac{1}{2}\left(2+\frac{9}{4}+3+\frac{17}{4}\right) = \frac{1}{2}\left(\frac{46}{4}\right) = \boxed{\frac{23}{4}}$

c) Is the result found in part b) an overestimate or underestimate? Carefully explain your reason. [1]

THIS IS AN $\boxed{\text{UNDERESTIMATE}}$ SEE THE PICTURE.

OR, SINCE f(x) IS INCREASING AND WE ARE USING LEFT ENDPOINTS, ALL RECTANGLES LIE UNDER THE GRAPH.

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3. If $\int_1^3 g(x)dx = 5$, $\int_1^0 g(x)dx = -2$ and $\int_2^3 g(x)dx = 2$ find $\int_0^2 (g(x)dx$.

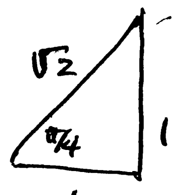
$$\int_0^2 g(x)\,dx = \int_0^1 g(x)\,dx + \int_1^2 g(x)\,dx$$
$$= \left(-\int_1^0 g(x)dx\right) + \left(\int_1^3 g(x)\,dx - \int_2^3 g(x)dx\right)$$
$$= (+2) + (5-2) = \boxed{5}$$

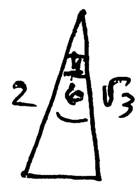
4. Calculate $\int_1^9 \frac{\sqrt{x}+1}{\sqrt{x}}$ dx.

$$\int_1^9 \frac{\sqrt{x}+1}{\sqrt{x}}\,dx = \int_1^9 \left(1+\frac{1}{\sqrt{x}}\right)dx = \int_1^9 \left(1+x^{-1/2}\right)dx$$
$$= x + 2x^{1/2}\Big|_1^9 \qquad 9^{1/2}=3$$
$$= (9+6)-(1+2)$$
$$= \boxed{12}$$

5. Calculate $\int_{\pi/6}^{\pi/4} \cos x \; dx$.

$$\int_{\pi/6}^{\pi/4} \cos x\,dx = \sin x\Big|_{\pi/6}^{\pi/4}$$





$$= \sin(\pi/4) - \sin(\pi/6)$$
$$= \boxed{\frac{1}{\sqrt{2}} - \frac{1}{2}}$$

EITHER

$$= \boxed{\frac{\sqrt{2}-1}{2}}$$

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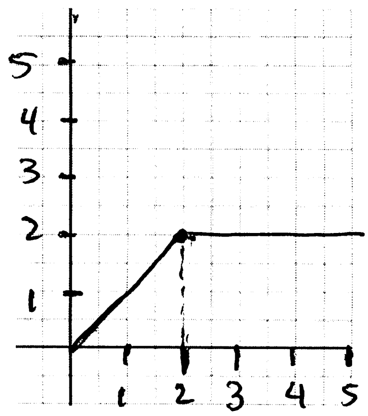
6. For the function $h(x) = \begin{cases} x \text{ for } x \le 2 \\ 2 \text{ for } x > 2 \end{cases}$

a) Sketch the graph from x= 0 to 5 on the axes provided. Pick an appropriate scale. [2]



b) Without using antiderivatives, find $\int_0^5 h(x)\,dx$. In other words use the geometric properties of the graph to evaluate the integral. [4]

$\int_0^2 h(x)\,dx$ IS THE AREA OF A TRIANGLE WITH BASE = 2, HT = 2 SO IT IS 2

$\int_2^5 h(x)dx$ IS THE AREA OF A RECTANGLE WITH BASE 3, HEIGHT 2, SO IT'S 6.

SO $\int_0^5 h(x)dx = 2 + 6 = 8$.

c) Calculate $\int_0^5 h(x)dx$ algebraically. Your answers to b) and c) should be equal. If they're not look for the error. [4]

$$\int_0^5 h(x)dx = \int_0^2 x\,dx + \int_2^5 2\,dx$$

$$= \frac{x^2}{2}\Big|_0^2 + 2x\Big|_2^5$$

$$= \left(\frac{4}{2} - 0\right) + (10 - 4)$$

$$= 2 + 6 = 8.$$

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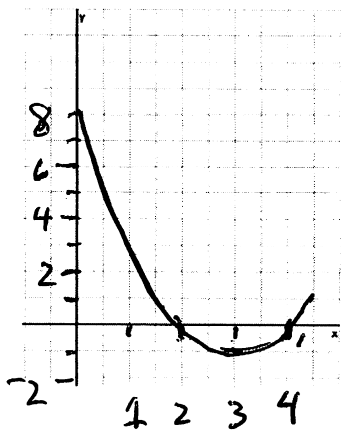
7. Find $\int x^3(x-4)$ dx. $= \int (x^4 - 4x^3)\,dx = \boxed{\frac{1}{5}x^5 - x^4 + C}$

8. The velocity function of a particle moving along a line (in m/s^2) is given by $v(t) = t^2 - 6t + 8$ on $0 \le x \le 4$.

a) Sketch the graph of the function on the given grid. [3]



b) Find the *displacement* over the given interval. [4]

$$\int_0^4 (t^2 - 6t + 8)\,dt$$

$$= \frac{t^3}{3} - 3t^2 + 8t \Big|_0^4$$

$$= \frac{64}{3} - 48 + 32 = 16/3$$

c) Set up an integral to find the *distance traveled* over the given interval but <u>do not</u> evaluate it. [3]

$$\int_0^4 |t^2 - 6t + 8|\,dt$$ IS THE TOTAL DISTANCE.

THIS IS $\int_0^2 (t^2 - 6t + 8)\,dt - \int_2^4 (t^2 - 6t + 8)\,dt$

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9. Calculate $\int_1^{e^3} \frac{2}{x}\, dx$.

$$\int_1^{e^3} \frac{2}{x}\,dx = 2\ln|x|\Big|_1^{e^3}$$
$$= 2\ln(e^3) - 2\ln(1)$$
$$= 2\cdot 3 - 0$$
$$= \boxed{6}$$

10. Find $\frac{d}{dx}\int_3^{e^{2x}} \sec(t)dt$.

$$= \sec(e^{2x})\cdot(2e^{2x})$$
$$= 2e^{2x}\sec(e^{2x})$$

YOU PROBABLY JUST MEMORIZED THAT

$$\int \sec t\,dt = \int \sec t\left(\frac{\sec t+\tan t}{\sec t+\tan t}\right)dt$$
$$= \int \frac{\sec^2 t + \sec t\tan t}{\sec t + \tan t}\,dt$$

LET $u = \sec t + \tan t$

SO $du = (\sec^2 t + \sec t\tan t)dt$

$$= \int \frac{du}{u} = \ln|u| + C$$
$$= \ln|\sec t + \tan t| + C$$

BUT THAT'S IRRELEVANT; USING THIS IS A MESS!