

**INSTRUCTIONS – PLEASE READ**

- 📵 Please turn off your cell phone and put it away.
- ⇨ Please write your name and your section number right now.
  - ⇨ This is a closed book exam. You are NOT allowed to use a calculator or any other electronic device or aid.
  - ⇨ The final has 8 problems worth a total of 150 points. Look over your test packet as soon as the exam begins. If you find any missing pages or problems please ask a proctor for another test booklet.
  - ⇨ Show your work. To receive full credit, your answers must be neatly written and logically organized. If you need more space, write on the back side of the preceding sheet, but be sure to label your work clearly. You do not need to simplify your answers unless explicitly instructed to do so.
  - ⇨ Academic integrity is expected of all Stony Brook University students at all times, whether in the presence or absence of members of the faculty.

PROBLEM	SCORE
1.	
2.	
3.	
4.	
5.	
6.	
7.	
8.	
<b>Total</b>	

<b>LEC 01</b>	MWF	10:00-10:53am	Joseph Adams
<b>R01</b>	F	1:00-1:53pm	Jaroslav Jaracz
<b>R02</b>	Tu	4:00-4:53pm	Charles Cifarelli
<b>R03</b>	Tu	1:00-1:53pm	Jaroslav Jaracz
<b>R04</b>	Th	8:30-9:23am	Alaa Abd-El-Hafez
<b>R05</b>	M	1:00-1:53pm	Thomas Rico
<b>R06</b>	M	9:00-9:53am	Zhuang Tao
<b>R07</b>	W	11:00-11:53am	Dyi-Shing Ou
<b>LEC 02</b>	TuTh	2:30-3:50pm	Raluca Tanase*
<b>R08</b>	Tu	4:00-4:53pm	Gaurish Telang
<b>R09</b>	Tu	1:00-1:53pm	Yuan Gao
<b>R10</b>	Th	1:00-1:53pm	Alaa Abd-El-Hafez
<b>R11</b>	F	1:00-1:53pm	Ruijie Yang
<b>R12</b>	W	12:00-12:53pm	Christopher Ianzano
<b>R13</b>	M	10:00-10:53am	Zhuang Tao
<b>R14</b>	M	12:00-12:53pm	Thomas Rico
<b>LEC 03</b>	MW	4:00-5:20pm	David Kahn
<b>R15</b>	W	9:00-9:53am	Ruijie Yang
<b>R16</b>	Tu	10:00-10:53am	Ying Chi
<b>R17</b>	W	10:00-10:53am	Ying Chi
<b>R18</b>	Th	4:00-4:53pm	Gaurish Telang
<b>R31</b>	W	5:30-6:23pm	Mariangela Ferraro
<b>R32</b>	M	5:30-6:23pm	Charles Cifarelli
<b>R33</b>	Tu	1:00-1:53pm	Yu Zeng

**Some trigonometric formulas that might be useful:**

$$\sin^2(x) + \cos^2(x) = 1$$

$$\sin(2x) = 2 \sin(x) \cos(x)$$

$$\tan^2(x) = \sec^2(x) - 1$$

$$\cos(2x) = 2 \cos^2(x) - 1 = \cos^2(x) - \sin^2(x)$$

**Problem 1.** (38 points) Evaluate the following integrals:

a)  $\int_{-1}^1 5x^3 + 3x + 1 \, dx$

b)  $\int x^2 e^{2x} \, dx$

c)  $\int \sin^9(x) \cos(x) \, dx$

*(Problem 1 continued)*

d)  $\int \frac{3x^2 - 2x + 3}{(x^2 - 1)(x^2 + 1)} dx$

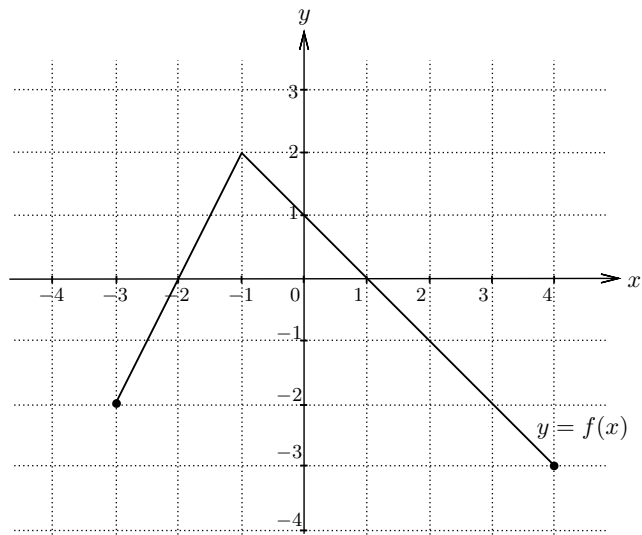
e)  $\int_0^4 \frac{dx}{(x - 2)^2}$

**Problem 2.** (18 points) Evaluate the following expressions:

a)  $\frac{d}{dx} \left( \int_3^{e^x} \arctan(\ln(t)) dt \right)$

b)  $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{n} \left( 1 + \frac{k}{n} \right)$

**Problem 3.** (10 points) Consider the function  $f(x)$  graphed below:



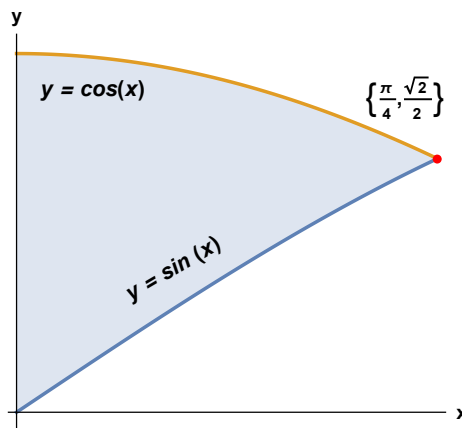
Now define a **new** function  $F(x) = \int_{-3}^x f(t) dt$  on the interval  $[-3, 4]$ .

For what value of  $x$  does  $F$  have a (global) maximum value? What is the maximum value?

Justify all your answers!

**Problem 4.** (30 points) The region  $R$  in the first quadrant bounded by  $y = \sin(x)$  and  $y = \cos(x)$  on the interval  $[0, \pi/4]$  is shown to the right.

a) Find the area of the region  $R$ .



b) Find the volume of the solid of revolution that results when  $R$  is revolved about the  $y$ -axis, using the Shell Method. Draw a typical cylindrical shell.

*(Problem 4 continued)*

- c) Find the volume of the solid of revolution that results when  $R$  is revolved about the  $x$ -axis, using the Disk/Washer Method. Draw a typical washer.

- d) Set up (but do not integrate!) the integral that gives the volume when  $R$  is revolved about the vertical line  $x = -1$ .

**Problem 5.** (12 points) Evaluate the integral  $\int \frac{x^3}{\sqrt{x^2-4}} dx$ . Simplify your final answer.

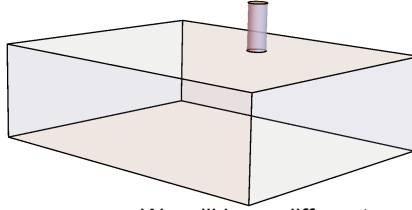


**Problem 6.** (18 points)

a) Calculate the arc length of the curve  $y = 2x^{3/2} + 1$  over the interval  $[0, \frac{1}{3}]$ .

b) Find the average value of the function  $y = \sin(x)e^{\cos(x)}$  over the interval  $[\frac{\pi}{2}, \pi]$ .

**Problem 7.** (14 points) A rectangular tank 5m long, 4m wide, and 1m deep is full of water. Find the work needed to pump the water out of the tank through a small spout at the top, with height  $1/2$  m. (The density of water is  $\rho = 1000\text{kg}/\text{m}^3$  and the constant of gravitational acceleration is  $g = 9.8\text{m}/\text{s}^2$ ).



This problem uses ideas we did not cover in our course. We will have different applications.

**Problem 8.** (10 points) Determine whether the following statements are true or false. Circle your response and give a brief explanation (a reason why it's true or an example where it fails).

a)  TRUE  FALSE If  $f$  is a continuous function on  $[a, b]$  such that  $\int_a^b f(x) dx = 0$ , then there exists at least one point  $x$  in  $(a, b)$  for which  $f(x) = 0$ .

b)  TRUE  FALSE Let  $f$  and  $g$  be two integrable functions on  $[a, b]$ . The definite integral  $\int_a^b (f(x) - g(x)) dx$  represents the area of the region between the graphs of  $f$  and  $g$ .