Reference Formulas: You may not need all of these.

$$cos^2x = \frac{1}{2}(1 + cos2x) \qquad sin^2x = \frac{1}{2}(1 - cos2x)$$

$$sin^2x + cos^2x = 1 \qquad tan^2x = sec^2x - 1$$

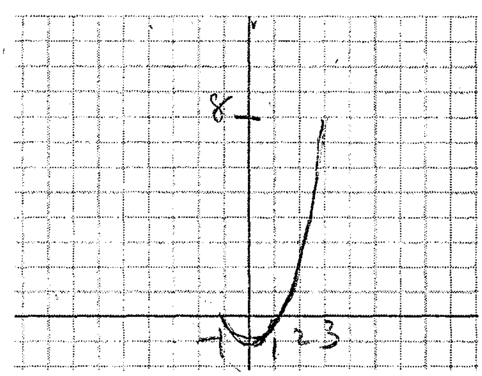
$$a^2\,cos^2\theta = a^2 - a^2sin^2\theta$$

$$a^2\,tan^2\theta = a^2sec^2\theta - a^2 \qquad a^2\,sec^2\theta = a^2tan^2\theta + a^2$$

Directions: Answer all questions in the space provided. You may use the blank backs of pages for scrap. No other paper is permitted. Show ALL relevant work. Calculators are not to be used. Circle your final answers. Show all work in the space provided. Be sure that you don't have answers to any question in more than one place. Answers without the required work will receive little or no credit. Simplify your answers. Each numbered question is worth 10 points. Note that the total number of points is 150.

1. For the function $f(x) = x^2 - 1$ do the following:

a) Sketch the graph for x = -1 to x = 3 on the axes at the right. Pick appropriate scales.



b) Approximate $\int_{-1}^{3} f(x)dx$ using a Riemann Sum using 4 sub-intervals and the LEFT endpoint of each sub-interval for as the sample points. Show the approximating rectangles on your graph. Use actual y values to compute the heights of the approximating rectangles. $\frac{b-a}{n} = \frac{4}{4} = 1$

$$\text{Sum} = \left((1) f(-1) + (1) f(0) + (1) f(1) + (1) f(2) \right)$$
$$= (1)(0) + (1)(-1) + (1)(0) + (1)(3)$$
$$= 2$$

2. Evaluate the integral: $\int x^2 e^{2x} dx$.

Let $u = x^2$ $dv = e^{2x}dx$

$du = 2x$ $v = \frac{e^{2x}}{2}$

$\int x^2 e^{2x}dx = \frac{x^2}{2}e^{2x} - \int xe^{2x}dx$

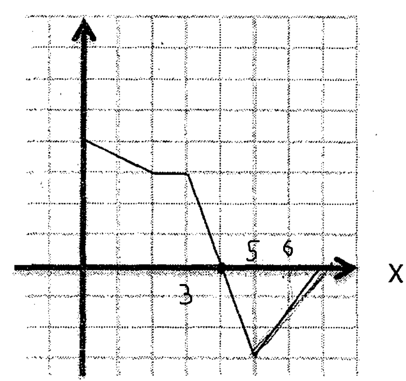
$u = x$ $dv = e^{2x}$

$du = dx$ $v = \frac{e^{2x}}{2}$

$= \frac{x^2}{2}e^{2x} - \left(\frac{x}{2}e^{2x} - \frac{1}{2}\int e^{2x}dx\right)$

$= \frac{x^2}{2}e^{2x} - \frac{x}{2}e^{2x} + \frac{1}{2}\int e^{2x}dx = \frac{x^2}{2}e^{2x} - \frac{x}{2}e^{2x} + \frac{1}{4}e^{2x} + C$

3. The graph of $f(x)$ is given at the right. <u>Each square is 1 unit by 1 unit</u>. If $g(x) = \int_0^x f(t)dt$ find the following:



a) $g(3)$ $= \int_0^3 f(t)dt = 10$

b) $g(5)$ $= \int_0^5 f(t)dt = 10$

c) $g(6)$ $= \int_0^6 f(t)dt = 8$

d) $g'(3)$ <u>Hint</u>: Fundamental Theorem

$g'(3) = f(3) = 3$

e) For what value of x does g have a maximum value? $x = 4$

4. Evaluate the integral $\int 2\cos^4 x \sin x$ dx.

Let $u = \cos x$

$du = -\sin x$

$= -2\int u^4 du = -\frac{2u^5}{5} = \frac{-2\cos^5 x}{5} + C$

5. Evaluate the integral $\displaystyle\int_{\sqrt{\frac{\pi}{4}}}^{\sqrt{\frac{\pi}{2}}} x\sin(x^2)\,dx$

$u = x^2$
$du = 2x\,dx$
$\frac{1}{2}du = x\,dx$

$$= \frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin u\,du = -\frac{1}{2}\cos u \Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} = -\frac{1}{2}\cos\frac{\pi}{2} + \frac{1}{2}\cos\frac{\pi}{4}$$

$$= \frac{\sqrt{2}}{4}$$

6. Write an integral that can be used to find the length of $y = \sqrt{2x+3}$ on the interval [1, 4] but do <u>NOT</u> evaluate it.

$$\frac{dy}{dx} = \frac{1}{2}(2x+3)^{-\frac{1}{2}} \cdot 2 = \frac{1}{\sqrt{2x+3}} \qquad \left(\frac{dy}{dx}\right)^2 = \frac{1}{2x+3}$$

$$\int_1^4 \sqrt{1 + \frac{1}{2x+3}}\,dx$$

7. Find the average value of the function $f(x) = e^x$ over the interval [0, ln 3].

$$\frac{1}{\ln 3 - 0}\int_0^{\ln 3} e^x\,dx = \frac{1}{\ln 3} e^x \Big|_0^{\ln 3} = \frac{1}{\ln 3}\left(e^{\ln 3} - e^0\right) = \frac{1}{\ln 3}(3-1)$$

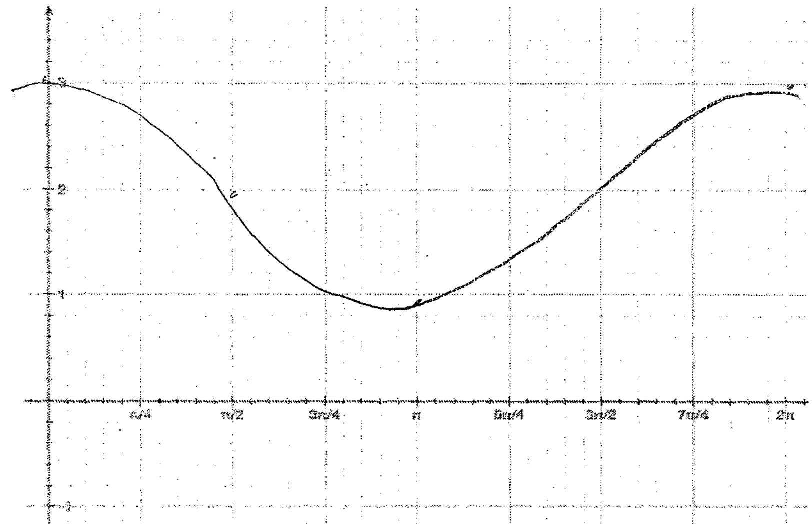
$$= \frac{2}{\ln 3}$$

8. Use the provided grids to do the following:

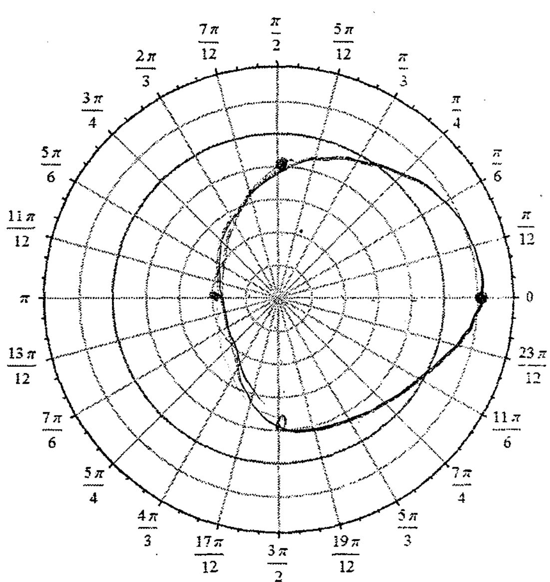
a) Sketch the graph of

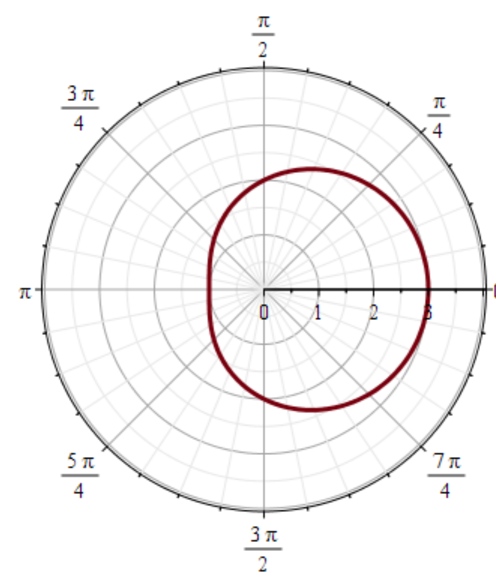
$y = \cos x + 2$

from x = 0 to 2π here →



b) Use your answer to part a) above to sketch the graph of the polar equation $r = \cos\theta + 2$ on the polar grid below. An extra grid is provided in case you need it. Use a scale in which each **2 rings = 1 unit**.





Here's a computer-generated version of the graph

c) Set up an integral that can be used to find the area inside the graph in your answer to question 8 b) above but do NOT evaluate it.

$$\frac{1}{2}\int_0^{2\pi} (\cos\theta + 2)^2 \, d\theta$$

9. Let R be the region between $y = x^2$ and $y = 2x$. Be sure to show all work.

a) Find the points of intersection of the two graphs.

$x^2 = 2x$

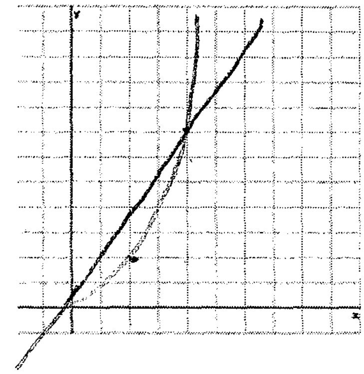
$x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0, 2$

$(0,0)$

$(2,4)$



b) Sketch the region R on the axes provided. Pick appropriate scales for x and y.

c) Use <u>cylindrical shells</u> to find the volume of the solid generated when the region R is revolved about the x-axis.

$y = x^2 \Rightarrow x = \sqrt{y}$ $\quad y = 2x \Rightarrow \frac{y}{2} = x$

$$V = 2\pi \int_0^4 y\left(\sqrt{y} - \frac{y}{2}\right) dy$$

$$= 2\pi \int_0^4 y^{\frac{3}{2}} - \frac{y^2}{2}\, dy = 2\pi\left(\frac{y^{\frac{5}{2}}}{\frac{5}{2}} - \frac{y^3}{6}\right)\Big|_0^4$$

$$= \left[\frac{4\pi}{5}(4)^{\frac{5}{2}} - \frac{\pi}{3}(4)^3\right] - 0 = \frac{128\pi}{5} - \frac{64\pi}{3}$$

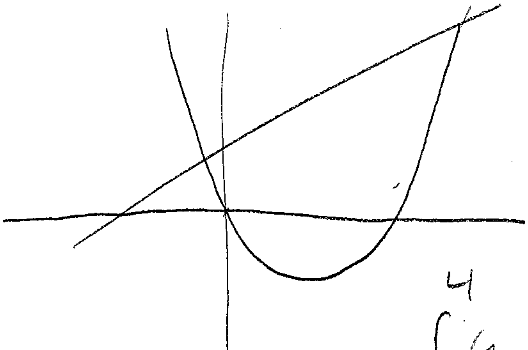
10. Evaluate the integral: $\int \frac{2x+3}{x^2+3x-4}\,dx.$

$u = x^2 + 3x - 4$

$du = 2x + 3\,dx$

$$= \int \frac{du}{u} = \ln|u| = \ln|x^2+3x-4| + C$$

11. Find the area bounded by $y = x^2 - 2x$ and $y = x + 4$.



$x^2 - 2x = x + 4$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x = -1, 4$

$$\int_{-1}^{4} (x+4) - (x^2 - 2x)\,dx = \int_{-1}^{4} -x^2 + 3x + 4\,dx$$

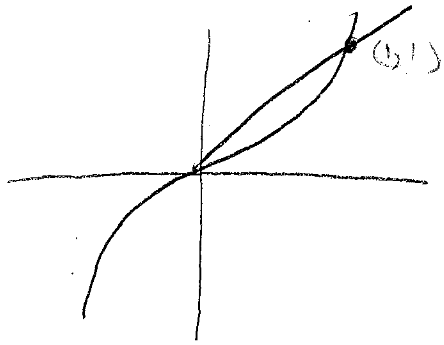
$$= -\frac{x^3}{3} + \frac{3x^2}{2} + 4x \Big|_{-1}^{4} = \left[\frac{-4^3}{3} + \frac{3(4^2)}{2} + 4(4)\right] - \left[\frac{-(-1)^3}{3} + \frac{3(-1)^2}{2} + 4(\right.$$

$$= -\frac{64}{3} + 24 + 16 - \frac{1}{3} - \frac{3}{2} + 4 = \frac{125}{6}$$

12. Find $\frac{d}{dx}\int_2^{x^2} \sqrt{1-t}\,dt$.

$$= \sqrt{1-x^2} \cdot 2x$$

13. Use disks or washers to find the volume when the region in the first quadrant bounded by $y = x$ and $y = x^3$, is revolved about the x-axis.



$$V = \pi \int_0^1 x^2 - (x^3)^2\, dx = \pi \int_0^1 x^2 - x^6\, dx = \pi \left(\frac{x^3}{3} - \frac{x^7}{7} \right) \Big|_0^1$$

$$= \pi \left(\frac{1}{3} - \frac{1}{7} \right) = \frac{4\pi}{21}$$

14. Evaluate the improper integral: $\int_2^3 \frac{1}{\sqrt{3-x}}\, dx$. State whether it converges or diverges.

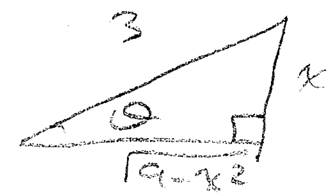
$$\lim_{a \to 3} \int_2^a (3-x)^{-\frac{1}{2}} dx = \lim_{a \to 3} \frac{-(3-x)^{\frac{1}{2}}}{\frac{1}{2}} \Big|_2^a$$

$$= \lim_{a \to 3} -2\sqrt{3-x} \Big|_2^a = \lim_{a \to 3} -2\sqrt{3-a} + 2 = 2$$

15. Evaluate the integral: $\int \frac{dx}{x^2\sqrt{9-x^2}}$ using the (See p. 2 for relevant formulas.) Leave your final answer in terms of x.

$x = 3\sin\theta$

$dx = 3\cos\theta\, d\theta$



$$= \int \frac{3\cos\theta\, d\theta}{9\sin^2\theta\sqrt{9-9\sin^2\theta}} = \int \frac{3\cos\theta\, d\theta}{9\sin^2\theta \cdot 3\sqrt{1-\sin^2\theta}}$$

$$= \int \frac{3\cos\theta\, d\theta}{9\sin^2\theta \cdot 3\sqrt{\cos^2\theta}} = \int \frac{d\theta}{9\sin^2\theta} = \frac{1}{9}\int \csc^2\theta\, d\theta$$

$$= -\frac{1}{9}\cot\theta = -\frac{1}{9}\left(\frac{\sqrt{9-x^2}}{x}\right) + C$$