Show using an ϵ, δ style argument (as was done in class) that

$$\lim_{(x,y)\longrightarrow(a,b)} 7x - y = 7a - b\,.$$

Hint: try modifying one of the examples we did in class.

Answer: First note that the function $f(x,y) = 7x - y$ has all of $\mathbb{R}^2$ as its domain. Let $\epsilon > 0$ be given. All we need to do is find a $\delta > 0$ so that

$$\mathrm{dist}((x,y),(a,b)) < \delta$$

implies

$$|f(x,y) - (7a-b)| < \epsilon\,.$$

We observe that

$$|f(x,y) - (7a-b)| = |7x - y - 7a + b| = |7(x-a) - (y-b)| \leq |7(x-a)| + |y-b| \leq \\ 7|x-a| + |y-b| \leq 7\mathrm{dist}((x,y),(a,b)) + \mathrm{dist}((x,y),(a,b)) = 8\mathrm{dist}((x,y),(a,b))\,.$$

Hence if

$$\mathrm{dist}((x,y),(a,b)) < \delta$$

then

$$|f(x,y) - (7a-b)| < 8\delta$$

and so by choosing $\delta = \frac{\epsilon}{8}$ we get

$$|f(x,y) - (7a-b)| < \epsilon$$

as required. $\square$

Note: It is wrong to say

$$|7(x-a) - (y-b)| \leq |7(x-a)| - |y-b|$$

since one may take as an example $a = b = 0,\ x = 1,\ y = -1$ and get

$$|7(x-a) - (y-b)| = 7 - (-1) = 8 \geq 6 = 7 - 1 = |7(x-a)| - |y-b|\,.$$