MAT320, Fall 2018

Practice Midterm II.

Problem 1. Let (s_n) be the sequence of partial sums of a series $\sum a_n$. If the series diverges, can (s_n) be

a) a bounded sequence?

b) a bounded increasing sequence?

Problem 2. Suppose a function $f : \mathbb{R} \to \mathbb{R}$ satisfies the following property

$$\forall \varepsilon > 0,\ \exists \delta > 0,\ \forall x, y \in \mathbb{R},\quad |x - y| < \delta \Rightarrow |f(x) - f(y)| < \varepsilon \qquad (*)$$

(in words: for any positive number ε there exists a positive number δ such that for any pair of real numbers x, y the inequality $|x - y| < \delta$ implies $|f(x) - f(y)| < \varepsilon$)

Does it then follow that f is continuous on $\mathbb{R}$?

Problem 3. Construct a continuous function $f : \mathbb{R} \to \mathbb{R}$ that does *not* satisfy property $(*)$ from the previous problem.

Problem 4. For points $\mathbf{x}, \mathbf{y} \in \mathbb{R}^2$ define $d(\mathbf{x}, \mathbf{y}) = |(x_1 - y_2)^2 - (x_2 - y_1)^2|$. Does d define a metric?

Problem 5. Show that in any metric space any compact set is closed.