Spring 2017 MAT 536, Complex Analysis
Instructor: Samuel Grushevsky
Homework #1, due in class Wed February 1

Problem 1. Find all complex solutions of the following equations: (a) $z^4 = -16$; (b) $z^4 + 4z^2 + 16 = 0$.

Problem 2. Prove that for any $a, b, c, d \in \mathbb{C}$ such that $ad - bc \neq 0$, the fractional-linear transformation

$$z \mapsto \frac{az + b}{cz + d}$$

defines a diffeomorphism $\mathbb{CP}^1 \to \mathbb{CP}^1$. *Hint: write down the inverse map.*

Problem 3. Prove that the image of any circle or a line in $\mathbb{C}$ under any fractional-linear transformation such as above is a circle or a line in $\mathbb{C}$ (and provide a correct interpretation for what this means if one point is mapped to $\infty \in \mathbb{CP}^1$).

Problem 4. For which fractional-linear transformations of the form above is the image of the (open) upper half-plane (a) equal to the upper half-plane; (b) contained in the upper half-plane?

Problem 5. Find the expression for the exponential function on the complex plane, starting from the basic properties of the exponential.

This means you should choose some set of "natural" properties that you want the exponential function $\exp : \mathbb{C} \to \mathbb{C}$ to satisfy, prove that these properties are satisfied by a unique function, and for any $z = x + iy$ write down $\operatorname{Re}\exp(z)$ and $\operatorname{Im}\exp(z)$ as functions of x and y.