MAT 320. HW due Oct 31, 2018

Do problems 18.12, 20.4, 20.8, 20.18 from the textbook.

Problem 1. If $f : \mathbb{R} \to \mathbb{R}$ is a *bijective* function such that the composition $f \circ f$ is continuous everywhere, does it necessarily follow that f is continuous everywhere?

Problem 2. Let the function $f : \mathbb{R} \to \mathbb{R}$ be defined by setting

$$f(x) = \begin{cases} x^2 & \text{if } x^2 \in \mathbb{Q} \\ 2 & \text{if } x^2 \notin \mathbb{Q}. \end{cases}$$

Determine the set of all $a \in \mathbb{R}$ such that the limit $\lim_{x \to a} f(x)$ exists.

Problem 3. Prove that there exists a unique *continuous* function

$$f : \mathbb{R} \to \mathbb{R}$$

such that $f(1) = 2$, and such that

$$f(x) \cdot f(y) = f(x + y)$$

for any $x, y \in \mathbb{R}$.

The following problem has been removed. Remember to submit Problem 3 from HW 7 instead, which has been postponed until now

~~**Problem 4.** Does there exist a function $f : \mathbb{R} \to \mathbb{R}$ such that $f(x) \cdot f(y) = f(x + y)$ for any $x, y \in \mathbb{R}$, but such that f is not everywhere continuous?~~