MAT200, Lecture 1 — Fall 2010

Midterm II practice problems

The midterm will be closed book, but the basic definitions, all axioms, and the main theorems would be given to you.

Problem 1. Let $\triangle ABC$ be a triangle. Prove that $m\angle A = m\angle B = m\angle C$ if and only if $|AB| = |BC| = |CA|$.

Problem 2. Use exterior angle inequality to prove that if l, m, n are distinct lines such that $l \perp m$, $l \perp n$, then $n \| m$ (for this problem you are only allowed to use chapters 1-5; in particular you are not allowed to use the sum of angles of a triangle).

Problem 3. Let A, B, C, D be distinct points such that $|AB| = |BC| = |CD| = |AD|$ and such that $M = \overline{AC} \cap \overline{BD}$. Show that $\triangle ABC = \triangle ADC$, $\triangle AMB = \triangle AMD$, and that $\overleftrightarrow{AC} \perp \overleftrightarrow{BD}$.

Problem 4. Let A and B be distinct points. Prove that the set of points C such that $|AC| = |BC|$ is a line. *Hint:* First show that there is a unique line ℓ through the mid-point of $\overline{AB}$ which is perpendicular to $\overleftrightarrow{AB}$. Then show that this line is what we want.

Problem 5. Suppose that our plane contained only four points A, B, C, D, any two a distance of one apart, and the six lines were $\overleftrightarrow{AB}$, $\overleftrightarrow{AC}$, $\overleftrightarrow{AD}$, $\overleftrightarrow{BD}$, $\overleftrightarrow{BC}$, $\overleftrightarrow{CD}$. Which of the axioms would this satisfy and which would it contradict? Explain why.

Problem 6. Let A, B, C be non-collinear points. Let D be a point of $\overline{BC}$. Without using the Crossbar theorem, prove that every point of the ray $\overrightarrow{AD}$ is in the interior of $\angle BAC$.