Stony Brook University
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Spring 2018

Problem Set 8

MAT 211: Linear Algebra

Homework Problems

8.1. *Diagonalizable or Not?* (15+5 points)

(a) Find the eigenvalues and corresponding eigenspaces of the matrix

$$A = \begin{bmatrix} -1 & 0 & 1 \\ -3 & 0 & 1 \\ -4 & 0 & 3 \end{bmatrix}.$$

(b) Write down the algebraic and geometric multiplicities of the eigenvalues of A. Is the matrix A diagonalizable?

8.2. *Diagonalization and Its Applications.* (15+5+5+10+10 points)

(a) Find the eigenvalues and corresponding eigenspaces of the matrix

$$A = \begin{bmatrix} 0 & 0 & 1/4 \\ 1 & 0 & -13/8 \\ 0 & 1 & 11/4 \end{bmatrix}.$$

(b) Write down the algebraic and geometric multiplicities of the eigenvalues, and argue that A is diagonalizable.

(c) Write down a basis $\mathcal{B} = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ of $\mathbb{R}^3$ consisting of eigenvectors of A. Using this, find an invertible matrix S such that $S^{-1}AS$ is a diagonal matrix.

(d) Find the coordinates of $\begin{bmatrix} 5 \\ -20 \\ 14 \end{bmatrix}$ with respect to the basis $\mathcal{B}$; i.e. write $\begin{bmatrix} 5 \\ -20 \\ 14 \end{bmatrix}$ as a linear combination of $\vec{v}_1, \vec{v}_2$, and $\vec{v}_3$.

(e) Compute $A^{1000} \begin{bmatrix} 5 \\ -20 \\ 14 \end{bmatrix}$.

8.3. *(Bonus question) Dynamics of Linear Maps* (10 points)

Let A be an $n \times n$ diagonalizable matrix with eigenvalues $\lambda_1, \cdots, \lambda_n$ such that $|\lambda_1| > |\lambda_i|$, for $i = 2, \cdots, n$. Moreover, assume that $\{\vec{v}_1, \cdots, \vec{v}_n\}$ is a basis of $\mathbb{R}^n$ consisting of eigenvectors of A with $A(\vec{v}_i) = \lambda_i \vec{v}_i$, for $i = 1, \cdots, n$. Finally, let $\vec{v} = c_1\vec{v}_1 + \cdots + c_n\vec{v}_n$, for some scalars $c_1, \cdots, c_n$. Prove that $\frac{1}{\lambda_1^n} A^n(\vec{v})$ converges to the vector $c_1\vec{v}_1$ as $n \to +\infty$.

Due Date: Wednesday, May 02.