

MAT 211: Linear Algebra

Homework Problems

7.1. Determinant and Invertibility (10 points)

Decide whether the following matrix is invertible by computing its determinant:

$$\begin{bmatrix} 2 & 3 & 0 & 2 \\ 4 & 3 & 2 & 1 \\ 6 & 0 & 0 & 3 \\ 7 & 0 & 0 & 4 \end{bmatrix}.$$

7.2. Determinant of Orthogonal Matrices. (5 points)

If A is an orthogonal matrix, what are the possible values of $\det(A)$?

7.3. Determinants of a Special Type of Matrices. (10 points)

Let P_n be the $n \times n$ matrix whose entries are all ones, except for zeros directly below the main diagonal; for example,

$$P_5 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix}.$$

Find the determinant of P_n .

7.4. Eigenvector of The Square of a Matrix. (10 points)

Let \vec{v} be an eigenvector of a matrix A with associated eigenvalue λ . Show that \vec{v} is an eigenvector of A^2 as well. What is the corresponding eigenvalue?

7.5. Finding Eigenvalues. (15 points)

Find all eigenvalues of the matrix

$$\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}.$$

7.6. (Bonus Problem) Invariant Line. (10 points)

Let A be a 3×3 matrix of real numbers. Show that there exists a line L (in \mathbb{R}^3) passing through the origin such that $A(L) \subset L$.

Due Date: Wednesday, April 25.