Stony Brook University
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Spring 2018

Problem Set 6

MAT 211: Linear Algebra

Homework Problems

6.1. *Orthogonal Complement* (8+7 points)

(a) Find a basis of the subspace W of $\mathbb{R}^4$ defined below:

$$W = \{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \in \mathbb{R}^4 : x_1 + x_2 + x_3 + x_4 = 0, x_1 - x_2 + x_3 - x_4 = 0, x_1 + 5x_2 + x_3 + 5x_4 = 0 \}.$$

What is the dimension of W?

(b) Find a basis of the orthogonal complement of W.

6.2. *Orthogonal Matrix.* (10 points)

Find a, b, c, d such that the matrix $\begin{bmatrix} a & b & 2/3 \\ 1/(3\sqrt{2}) & c & 2/3 \\ -4/(3\sqrt{2}) & 0 & d \end{bmatrix}$ is orthogonal.

6.3. *Gram-Schmidt Orthonormalization.* (8+7 points)

(a) Apply Gram-Schmidt orthonormalization on the vectors $\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} 18 \\ 0 \\ 0 \end{bmatrix}$.

(b) Using the result of part (a), find the QR factorization of the matrix $\begin{bmatrix} 2 & -2 & 18 \\ 2 & 1 & 0 \\ 1 & 2 & 0 \end{bmatrix}$.

6.4. *Orthogonal Projection.* (10 points)

Find the orthogonal projection of $\begin{bmatrix} 49 \\ 49 \\ 49 \end{bmatrix}$ onto the subspace of $\mathbb{R}^3$ spanned by $\begin{bmatrix} 2 \\ 3 \\ 6 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ -6 \\ 2 \end{bmatrix}$.

Due Date: Wednesday, April 11.