Stony Brook University
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MAT 211: Linear Algebra

Homework Problems

5.1. *Linear Independence-I* (5+5 points)

Decide whether the following sets of vectors are linearly independent.

(a) $\begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}1\\3\\6\end{bmatrix}.$

(b) $\begin{bmatrix}1\\1\\1\\1\end{bmatrix}, \begin{bmatrix}1\\2\\3\\4\end{bmatrix}, \begin{bmatrix}1\\4\\7\\10\end{bmatrix}.$

5.2. *Linear Independence-II.* (10 points)

Suppose that u_1, u_2 and u_3 are linearly independent vectors in $\mathbb{R}^n$. Show that the vectors $u_1 + u_2$, $u_2 + u_3$ and $u_3 + u_1$ are also linearly independent.

5.3. *Coordinates of Vectors.* (10 points)

Show that $\mathcal{B} = \{ \begin{bmatrix}1\\0\\-1\end{bmatrix}, \begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}1\\0\\0\end{bmatrix} \}$ is a basis of $\mathbb{R}^3$. What are the coordinates of the vector $\begin{bmatrix}x\\y\\z\end{bmatrix}$ with respect to the ordered basis $\mathcal{B}$?

5.4. *Basis and Dimension.* (10+15 points)

(a) For which value(s) of the constant k do the vectors

$$\begin{bmatrix}1\\0\\0\\2\end{bmatrix}, \begin{bmatrix}0\\1\\0\\3\end{bmatrix}, \begin{bmatrix}0\\0\\1\\4\end{bmatrix}, \begin{bmatrix}2\\3\\4\\k\end{bmatrix}$$

form a basis of $\mathbb{R}^4$?

(b) Find a basis of the subspace W of $\mathbb{R}^5$ defined below:

$$W = \{ \begin{bmatrix}x_1\\x_2\\x_3\\x_4\\x_5\end{bmatrix} \in \mathbb{R}^5 : 2x_1 - x_2 + \frac{4}{3}x_3 - x_4 = 0, x_1 + \frac{2}{3}x_3 - x_5 = 0, 9x_1 - 3x_2 + 6x_3 - 3x_4 - 3x_5 = 0 \}.$$

What is the dimension of W?

5.5. *Matrix of a Linear Map.* (10 points)

Find the matrix B of the linear transformation $T(\vec{u}) = A\vec{u}$ with respect to the basis $\mathcal{B} = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$, where

$$A = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -2 & 2 \\ 3 & -9 & 6 \end{bmatrix},$$

and

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix}.$$

5.6. *Finding Basis with Prescribed Properties.* (10 points)

Find a basis $\mathcal{B}$ of $\mathbb{R}^2$ such that the coordinates of the vectors $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$ with respect to $\mathcal{B}$ are $\begin{bmatrix} 3 \\ 5 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ respectively.

Due Date: Wednesday, March 28.