Stony Brook University
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Spring 2018

Problem Set 3

MAT 211: Linear Algebra

Homework Problems

3.1. *Matrix Multiplication.* (10 points)

Compute the following matrix product.

$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

3.2. *Commuting Matrices.* (10 points)

Find all 3×3 matrices A such that $AB = BA$, where $B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$.

3.3. *Geometric Interpretation of Matrices.* (5 points)

Find a 2×2 matrix A such that $A^5 = \begin{bmatrix} 1 & 10 \\ 0 & 1 \end{bmatrix}$.

3.4. *Computing The Inverse of a Matrix.* (10+10 points)

Decide whether the following matrices are invertible. If they are, find the inverse.

$$\begin{bmatrix} 1 & 2 & 2 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 2 & 1 & 0 \\ 4 & 3 & 2 & 1 \end{bmatrix}$$

3.5. *Conditions for Invertibility.* (10 points)

For which values of the constants a, b, and c is the following matrix invertible?

$$\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$$

3.6. *Invertible Transformations.* (15 points)

Which of the following linear transformations $T : \mathbb{R}^3 \to \mathbb{R}^3$ are invertible? Find the inverse if it exists.

(a) Reflection about a plane,

(b) Orthogonal projection onto a plane,

(c) Scaling by a factor of 5 [i.e., $T(u) = 5u$, for all vectors u in $\mathbb{R}^3$].

3.7. *(Bonus problem) Classifying Linear Transformations of Order Two.* (15 points)

Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that $T^2 = \text{Id}$. Prove that one of the following conditions holds.

- T is the identity transformation,
- T preserves every straight line through the origin (i.e. T maps every straight line through $(0, 0)$ to itself),
- T fixes exactly two straight lines through the origin.

Due Date: Wednesday, February 21.