Stony Brook University
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Problem Set 2

MAT 211: Linear Algebra

Homework Problems

Recall that $\mathbb{R}^n$ is the collection of all column vectors (or coordinate vectors) of size n; i.e.

$$\mathbb{R}^n = \{ \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} : x_1, x_2, \cdots, x_n \in \mathbb{R} \}$$

.

The coordinate vectors $e_1 = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \cdots, e_n = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$ can be thought of as building blocks of $\mathbb{R}^n$ (the vector e_k has a 1 at the k-th position and 0 everywhere else). Indeed, we can write any coordinate vector $\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ as $\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = x_1 e_1 + x_2 e_2 + \cdots + x_n e_n$.

A map $T : \mathbb{R}^n \to \mathbb{R}^m$ is called *linear* if

(a) $T(u + v) = T(u) + T(v)$, for all $u, v \in \mathbb{R}^n$, and

(b) $T(cu) = cT(u)$, for all $c \in \mathbb{R}$ and $u \in \mathbb{R}^n$.

We saw that a map $T : \mathbb{R}^n \to \mathbb{R}^m$ is linear (according to the definition above) if and only if there exists an $m \times n$ matrix A such that $T\left(\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}\right) = A \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$. Moreover, the columns of A are $T(e_1), T(e_2), \cdots$, and $T(e_n)$. The matrix A is called the matrix of the linear map T.

2.1. *Matrix of Linear Maps.* (15+15 points)

(a) Let $T : \mathbb{R}^3 \to \mathbb{R}^4$ be defined as $T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 9x_1 + 3x_2 - 3x_3 \\ 2x_1 - 9x_2 + x_3 \\ 4x_1 - 9x_2 - 2x_3 \\ 5x_1 + x_2 + 5x_3 \end{bmatrix}$. Is T a linear map? If so, find the matrix of T.

(b) Consider the transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ given by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = x_1 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} - x_2 \begin{bmatrix} 4 \\ 5 \\ -6 \end{bmatrix}$. Is this transformation linear? If so, find its matrix.

2.2. *Orthogonal Projection onto a Line.* (10 points)

Let L be the line in $\mathbb{R}^3$ that consists of all scalar multiples of the vector $\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$. Find the orthogonal projection of the vector $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ onto L.

2.3. *Reflection about a Line.* (10 points)

Let L be the line in $\mathbb{R}^3$ that consists of all scalar multiples of the vector $\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$. Find the reflection of the vector $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ about L.

2.4. *Rotation as a Linear Map.* (10 points)

Find the rotation matrix that transforms $\begin{bmatrix} 0 \\ 5 \end{bmatrix}$ to $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$.

Due Date: Wednesday, February 14.