

MAT 211: Linear Algebra

Homework Problems

1.1. *Elimination Method.* (10 points)

Solve the following system of linear equations using the method of elimination:

$$x + 2y + 3z = 1,$$

$$2x + 4y + 7z = 2,$$

$$3x + 7y + 11z = 8.$$

1.2. *Dependence on a Parameter.* (20 points)

Consider the linear system

$$x + y - z = -2,$$

$$3x - 5y + 13z = 18,$$

$$x - 2y + 5z = k,$$

where k is an arbitrary real number.

- For which value(s) of k does this system have one or infinitely many solutions?
- For each value of k you found in the previous part, how many solutions does the system have?
- Find all solutions for each value of k obtained in the first part.

1.3. *Application to Geometry.* (15 points)

Find a , b , and c such that the ellipse $ax^2 + bxy + cy^2 = 1$ passes through the points $(1, 2)$, $(2, 2)$, and $(3, 1)$.

1.4. *Gauss-Jordan.* (10+10 points)

Solve the following systems of linear equations using Gauss-Jordan elimination (i.e. write down the augmented matrix, and put it in RREF):

(a) $3x + 11y + 19z = -2,$

$$7x + 23y + 39z = 10,$$

$$-4x - 3y - 2z = 6.$$

(b) $x_1 + 2x_3 + 4x_4 = -8,$

$$x_2 - 3x_3 - x_4 = 6,$$

$$3x_1 + 4x_2 - 6x_3 + 8x_4 = 0,$$

$$-x_2 + 3x_3 + 4x_4 = -12.$$

1.5. (*Bonus problem*) *Integral Solutions.* (10 points)

Consider the system

$$2x + y = C,$$

$$3y + z = C,$$

$$x + 4z = C,$$

where C is a constant. Find the smallest positive integer C such that x , y , and z are all integers.

Due Date: Wednesday, February 7.