Getting Started with Mathematica

Basic calculations and variables

Mathematica is a powerful tool for doing mathematics. It can handle both symbolic and numeric expressions. In the simplest case you can use it just like a calculator: you type in questions, and Mathematica prints back answers.

\[ 27 \times 5^3 \]

You can also use variables or put several commands, separated by semicolons, on one line. A semicolon also has the effect of suppressing the corresponding output. To write the power as a superscript, go to Palettes->Basic Math Assistant.

\[ x = 5; \quad y = 27; \quad y * x^3 \]

We can also write each command on a separate line (enter/return) within the same cell.

\[ x = 5 \]
\[ y = 27; \]
\[ y * x^3 \]

Writing several commands not separated by semicolons on the same line can lead to different results. By default, a space between two expressions is interpreted as multiplication! Try writing something like \( x=5 \) \( 27+x^3! \). What do you get?

You might have noticed that Mathematica assigns numbers to every command you input and to the corresponding output. In particular, all results are stored and you can use them in any following calculations as long as you do not quit Mathematica (more precisely the kernel).

\[ \text{Out[1]} = 3 \]

In addition, the shortcut \% can be used to refer to the previous result.

\[ \% / 3 \]

Moreover, Mathematica is case sensitive.

\[ X + x \]

Symbolic versus Numeric Computations

\[ a = 2^{10} + \frac{1}{2} \]
\[ b = \text{N}[a] \]
\[ c = \text{N}[a, 10] \]
\[ d = 2^{10} + \frac{1}{2} \sqrt{2} \cos \pi / 2 \]

You can use the \text{N[\text{\ldots}]\space{\ldots}} command to get a numeric answer. In addition, this allows to give the number of digits you want as an optional argument.

**Exercise 1:** Use the sinus function \text{Sin[\ldots]} to compute sinus of angle 2, with precision of one hundred decimal points

Matrices and Linear Algebra

The Mathematica front end provides an Insert->Table/Matrix submenu for creating and editing arrays with any specified number of rows and columns. Once you have such an array, you can edit it to fill in whatever elements you want.

Some examples of vectors and matrices:

Mathematica treats a matrix like a list of lists. To see it in standard matrix form, use the command MatrixForm[\ldots]

\[ A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix} \]

\[ \text{IdentityMatrix[3]} \]
\[ \text{MatrixForm[\ldots]} \]

A vector can be given as a list.

\[ w = \{1, 2, 3\} \]
\[ \text{MatrixForm[w]} \]

We can also use the Insert->Table/Matrix submenu to create a column vector with a given number of elements, as a matrix with \( n \) rows and one column. If we use this template, the vector is interpreted as a list of lists.

\[ v = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \]
Matrix and Vector Operations


We can perform matrix addition $A+B$ and scalar multiplication $3A$ in the usual way. However, to do matrix multiplication we cannot use the symbol $\cdot$, instead we have to use a dot!

$$A \cdot A; \text{MatrixForm}[A]$$

$A.A$  \text{MatrixForm}[A]$\text{Transpose}[A]$  \text{MatrixForm}[A.w]$\text{MatrixForm}[A.v]$

Transpose $[\text{matrix}]$ -- Computes the transpose
MatrixPower $[\text{matrix}, \text{power}]$ -- For square matrices, we can compute the power of a matrix $A^n$
Eigensystem $[\text{matrix}]$ -- Finds both the eigenvalues and a complete linearly independent set of
eigenvectors for each eigenvalue.
Eigenvectors $[\text{matrix}]$ -- Finds the eigenvectors of a square matrix

$B = \{(1, 1), (1, 1)\}; \text{MatrixPower}[B, 3]$

Exercise 2: Find the eigenvalues and eigenvectors of matrix $B$.

Changing Elements

To access different matrix or vector elements use $[\ldots]$.

$w = \{10, 20, 30\}; w[[1]]$

$10$

$B = \{(10, 20), (30, 40)\}$  \text{MatrixForm}[B]$\text{Transpose}[A]$  \text{MatrixForm}[A.w]$\text{MatrixForm}[A.v]$

To change elements of a matrix or vector:

$w[[1]] = 100; w$

$(100, 20, 30)$

Length $[\ldots]$ returns the number of elements in a list.

Exercise 3: What is the length of $w$? How about the length of $B$?

Exercise 4: Write a Mathematica command that gives the sum of the elements of the vector $w$.

Other ways of generating a matrix:

Table[f, $\{i, \text{ilm} \}$, $\{j, \text{ijn}]$ builds an $m\times n$ matrix by evaluating the function $f$ with arguments $i$ and $j$, where $i$ ranges from 1 to $m$ and $j$ ranges from 1 to $n$. The lower bound is implicitly 1, so we only specify the upper bound.

The following commands generate random $4 \times 4$ matrix with entries between 0 and 1:

$Rn = \text{Table}[\text{Random}[]], \{i, 4\}, \{j, 4\}]$

$(0.727347, 0.0991694, 0.716267, 0.20713), (0.597192, 0.089866, 0.550936, 0.0729925), (0.24518, 0.516913, 0.86244, 0.519886), (0.823346, 0.798585, 0.986709, 0.0177835)$

$R1 = \text{Table}[\text{RandomInteger}[]], \{i, 4\}, \{j, 4\}]$

$\{(1, 0, 1, 0), (3, 1, 0, 1), (1, 1, 1, 1), (0, 0, 1, 0)\}$

We may also want to define our own function $f$.

$f[_, j_] := i + j$

In this expression the underscore represents a pattern. A single underscore will match a single expression (argument) and $x$ is the name of the pattern (it can be used to refer to the matched expression on the right-hand side). The colon tells Mathematica not to evaluate the right-hand side. It will only be evaluated when you invoke the function. This is why you get no output. Now you can use your function:
Try to assign some values to i and j, then compute f[2, 3]. What do you notice? Does f use the global or the local values of i and j when computing f[2, 3]?

Warning: Do not omit the colon when defining functions!

Can you explain this result? What is the function g? You can look at the definition of a function by using a question mark.

To clear the definition of function g, use the command Clear[g].

We can then use function f to generate a 4x4 matrix whose i,j entry is i+j.

Exercise 5: Generate a 5x5 matrix C
with C[i,j] = i if i>=j and C[i,j] = 0 otherwise

The syntax of the "If" command in Mathematica is
If[condition, t, f] - the "If" command returns t if condition evaluates to true and f if condition evaluates to false.

If[3 > 4, 10, 5]
If[3 < 4, 10, 5]