MAT 331: Homework 3

Problem 3.1

The replacement operator /. (slash-dot) applies a transformation rule to an expression. If you give a list of rules, each rule will be tried once on each part of the expression. If you give a list of lists of rules, you get a list of results; each sublist is treated like an independent set of rules.

\[ x^2 + x^3 + y^2 + z \/. \{x_\to 2\} \]

\[ x^2 + x^3 + y^2 + z \/. \{x \to 2, y \to 3, z \to 1\} \]

\[ x^2 + x^3 + y^2 + z \/. \{(x \to 2), (x \to 2, y \to 3, z \to 1), (x \to 0, z \to 0)\} \]

\[ x^2 + x^3 + y^2 + z \/. \{x_\to n_\to a\} \]

1. Write a replacement rule that when applied to the expression \( f[x] + g[x] \) outputs \( \text{Sin}[x] + \text{Cos}[3] \).
2. Write a transformation rule that replaces any expression of the form \( \text{Function}[	ext{variable}] \) with \( \text{Cos}[3] \).

Problem 3.2

Mathematica can solve various kinds of equations, symbolically or numerically. The result will be displayed as a list of transformation rules. We can then use the replacement operator /. (slash-dot) to apply these rules to any given expression.

Consider now the cubic polynomial \( x^3 + ax^2 + bx + c \). Use Mathematica to find the roots \( x_1, x_2 \) and \( x_3 \). Then use transformation rules to compute the three symmetric expressions \( x_1 x_2 + x_3, x_1 x_2 + x_2 x_3 + x_3 x_1 \) and \( x_1 x_2 x_3 \). Use \( \text{Simplify}[...] \) to simplify the computations. What can you conclude about the relation between the three expressions and the coefficients of the polynomial?

Problem 3.3

Consider the differential equation \( y'(x) = 1 + y(x) \).
1. Use \( \text{VectorPlot}[...] \) and \( \text{StreamPlot}[...] \) to plot the vector field of the differential equation. Try several display options (i.e. change the size of the arrows, make the picture larger, change the colors, etc.) to see which one gives the most accurate picture.
2. Next use \( \text{DSolve}[...] \) to solve the differential equation \( y'(x) = 1 + y(x) \), with initial condition \( y(0)=1 \). The result will be a list of transformation rules. Define a function \( \text{YSol}[x_] \) that returns the solution found by \( \text{DSolve}[...] \). Evaluate \( \text{YSol} \) numerically at \( x=0 \) and \( x=0.1 \). Then plot the function \( \text{YSol} \).
3. Check that \( \text{YSol} \) is indeed a solution of the differential equation \( y'(x) = 1 + y(x) \) by using Mathematica to compute the derivative of the function \( \text{YSol} \).
4. Now go back to the picture that you have obtained in part 1, using StreamPlot[..]. Color the solution of the differential equation \( y'(x) = 1+y(x) \) corresponding to the initial condition \( y(0)=1 \) on top of the stream plot picture.