MAT 341: Applied Real Analysis – Fall 2015

HW9 – Comments

Sec. 3.3 – Problem 1: The problem is asking you to find some values of $u(x,t)$ such that

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}, \qquad 0 < x < a, \qquad t > 0;$$
$$u(0,t) = 0, \quad u(a,t) = 0, \qquad t > 0;$$
$$u(x,0) = f(x), \qquad t > 0;$$
$$\frac{\partial u}{\partial x}(x,0) = 0, \quad 0 < x < a.$$

where $f(x)$ has the following equation:

$$f(x) = \begin{cases} \dfrac{2h}{a}x & \text{if} \quad 0 \le x \le \frac{a}{2} \\ -\dfrac{2h}{a}x + 2h & \text{if} \quad \frac{a}{2} < x \le a. \end{cases}$$

You then need to write a table with the values $u(x,t)$ at the required times, such as $u(0.25a, 0.2a/c)$. The solution $u(x,t)$ is written in Equation 13, but without the function G_e. **Note:** In the textbook, $\overline{f}_o$ means an odd periodic extension of f, while $\overline{G}_e$ means an even periodic extension of G.

Sec. 3.3 – Problem 2: You fix time $t = 0, 0.2a/c, 0.4a/c, 0.8a/c, 1.4a/c$ and you sketch 5 graphs of $u(x,t)$. For example, you need to sketch the graph of $u(x, 0.4a/c)$ as a function of x. You may assume $a = 1$ if it helps. The graphs should look like Figure 3 from Section 3.2.

Sec. 3.3 – Problem 5: The solution $u(x,t)$ verifies the PDE:

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}, \qquad 0 < x < a, \qquad t > 0;$$
$$u(0,t) = 0, \quad u(a,t) = 0, \qquad t > 0;$$
$$u(x,0) = 0, \qquad 0 < x < a;$$
$$\frac{\partial u}{\partial t}(x,0) = \alpha c, \quad 0 < x < a.$$

where α is just a constant, unrelated to a.

Sec. 4.1 – Problem 2: The sketch of the surfaces should look like the graphs below.

Regarding the boundary conditions: you have to evaluate $u(x,y)$, $\frac{\partial u}{\partial x}(x,y)$ and $\frac{\partial u}{\partial y}(x,y)$ at the given values. For example, if $u(x,y) = xy$ then $u(0,b) = 0$ and $u_x(0,b) = b$, $u_y(0,b) = 0$.

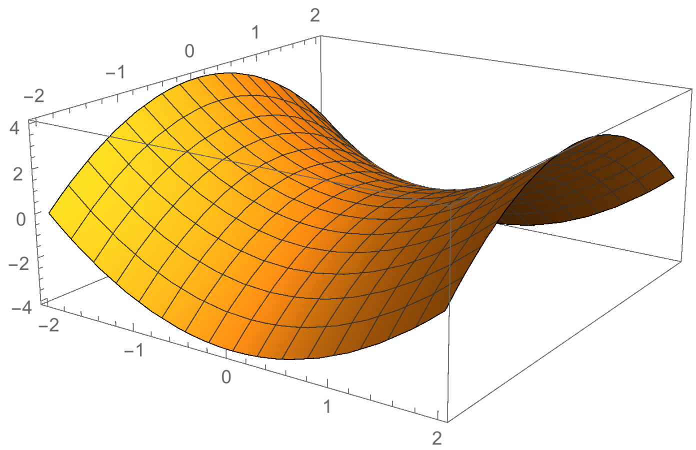

Figure 1: A sketch of the surface $z = x^2 - y^2$.

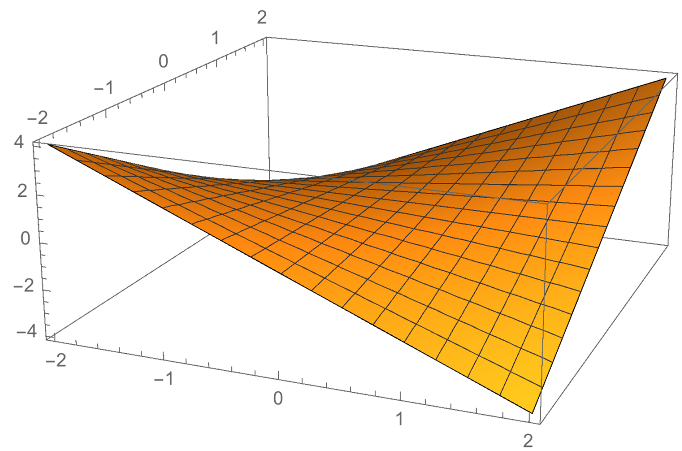

Figure 2: A sketch of the surface $z = xy$.

Sec. 4.2 – Problem 5: You are asked to solve the following PDE:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \qquad 0 < x < 1, \quad 0 < y < b;$$

$$u(0, y) = 0, \quad u(1, y) = 0, \qquad 0 < y < b;$$

$$u(x, 0) = 0, \quad u(x, b) = \sin(3\pi x), \qquad 0 < x < 1;$$

You may assume that b is any constant. However, once you reach a formula for $u(x, y)$ as in Equation 9 (page 266) there is no need to compute the coefficients, simply use the fact that you already have $\sin(3\pi x)$ as a Fourier series and look for the coefficient of $n = 3$ (the rest are all zeros). To sketch the level curves, one has to do as in Figure 2, page 268 (see next page).

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(*Problem 4.2:5 Sketch of level curve. *)
a = 1;
b = 0.5;
const = 0.1;
ContourPlot[Sinh[3*π*y]/Sinh[3*π*b] Sin[3 π*x] == const, {x, 0, a}, {y, 0, b}]
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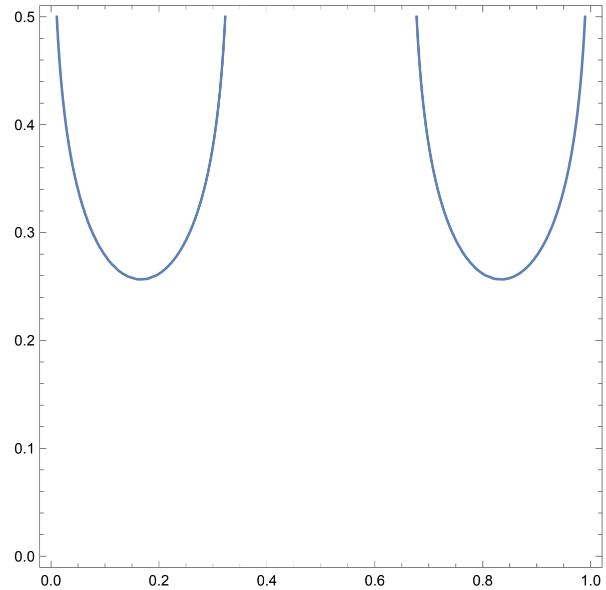


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Plot3D[Sinh[3*π*y]/Sinh[3*π*b] Sin[3 π*x], {x, 0, a}, {y, 0, b}]
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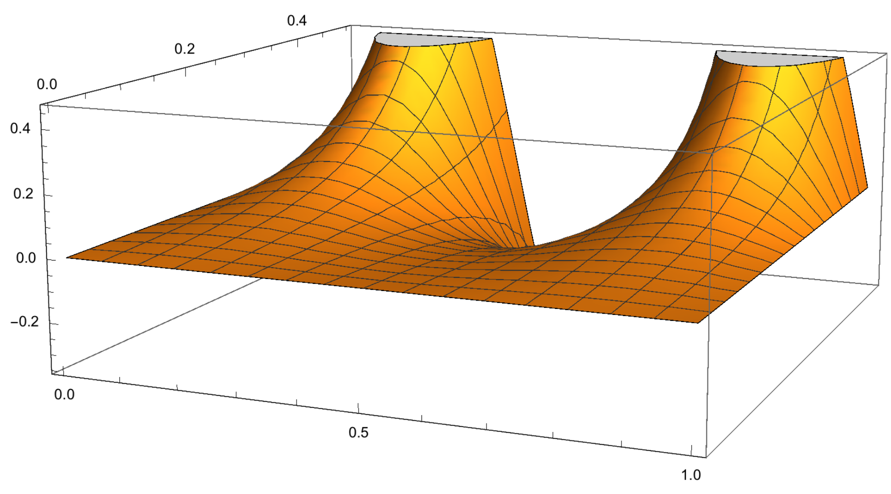

Figure 3: Top: Level curves $u(x, y) = const$ drawn in Mathematica. Bottom: The surface $z = u(x, y)$. The level curves are obtained by cutting the level surface by a plane transversly.

Sec. 4.2 – Problem 6: You are asked to solve the following PDE:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \qquad 0 < x < a, \quad 0 < y < b;$$

$$u(0,y) = 0, \quad u(a,y) = 1, \quad 0 < y < b;$$

$$u(x,0) = 0, \quad u(x,b) = 0, \quad 0 < x < a;$$