Math 312/ AMS 351 (Fall '17)
Midterm 1
September 26, 2017

1. Prove that if x is not equal to 1 and n is any positive integer
   then
   $$1 + x + x^2 + \cdots + x^n = \frac{x^{n+1} - 1}{x - 1}.$$

2. a) Prove that no number of the form $4k+3$ can be written as a sum of two squares.

   b) Show that a number n is divisible by 9 iff the sum of its digits is divisible by 9. Give an example of this criterion for a number with 4 digits.

3. Show that there exist infinitely many primes of the form $3k+2$.

4. Solve the following equations

i) $3x + 2 \equiv x + 4 \mod 9$

ii) $6x + 2 \equiv 4 \mod 9$

iii) $x \equiv 2 \mod 5$ and $x \equiv 3 \mod 7$

5.   i) Compute 7^{66} mod 120.

ii) Prove that for any a and $n > 2$, the last two digits of a^n can not be 30.