Math 313 (Fall '09)

Homework 4

due October 8

The following exercises refer to the textbook (the seventh edition).

- Ch9: 11, 14, 25
- Ch11: 10, 12

Sample Midterm

1. Which of the following sets are subgroups of $GL(2, \mathbb{R})$:
   
   i) $H_1 = \{ X \in GL(2, \mathbb{R}) \mid \det X = 1 \}$;
   
   ii) $H_2 = \{ X \in GL(2, \mathbb{R}) \mid \det X = -1 \}$;
   
   iii) $H_3 = \{ X \in GL(2, \mathbb{R}) \mid \text{the entries in } X \text{ belong to } \mathbb{Z} \}$;
   
   iv) $H_4 = \{ X = \begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix} \mid a, b \in \mathbb{R} \}$

   Explain!

2. What are the possible order of permutations in $S_7$. How many permutations of order 8 and 10 respectively are in $S_7$? What is the number of even permutations of order 6 in $S_7$?

3. Classify all groups with 8 elements.

   **Hint:** As a first step you should list all groups with 8 elements that you know. Pay attention to the maximal order that occurs in each case. Then consider the abelian case. Finally, discuss the non-abelian case.

4. Let $G$ be a cyclic group. Prove that
   
   i) any subgroup $H$ of $G$ is cyclic;
   
   ii) any factor group $G/H$ is cyclic.

   Additionally, give an example to show that it does not suffice to know that $H$ and $G/H$ are cyclic, to conclude that $G$ is cyclic.

5. Show that a group of order 33 must have an element of order 3.

6. The set $\{1, 9, 16, 22, 29, 53, 74, 79, 81\}$ is a group under multiplication modulo 91. Determine the isomorphism class of this group.