1. Let \( \ell \) be a line, and \( P \notin \ell \) a point.
   i) What is the locus of points at fixed distance \( x \) from \( P \)?
   ii) What is the locus of points at fixed distance \( y \) from \( \ell \)?
   iii) Find a point \( Q \) which is at distance \( x \) from \( P \) and distance \( y \) from \( \ell \).
   iv) How many points \( Q \) are at distance \( x \) from \( P \) and distance \( y \) from \( \ell \)? (N.B. here you should get different answers depending on the distance between \( \ell \) and \( P \))

2. Given a segment \( AB \) and a point \( M \) on this segment:
   i) Construct a point \( P \) such that \( \angle APB = 60^\circ \). What is the locus of points \( P \) with this property?
   ii) Construct a point \( P \) such that \( \angle APB = 60^\circ \) and \( PM \) is the bisector of angle \( \angle P \).

3. (This exercise tests the use of sine/cosine laws)
   1) Compute \( \sin 60^\circ \) and \( \cos 60^\circ \) (Hint: use an equilateral triangle).
   2) Given a triangle \( ABC \) such that \( \angle BAC = 60^\circ \), \( AB = 2 \), \( AC = 5 \), compute \( BC \) and then the other two angles (i.e. \( \sin \) or \( \cos \) of those angles).
   3) Decide if the angles at \( B \) and \( C \) are acute or obtuse. (Before you do any computation, which angle could be obtuse - justify)
   4) Compute the distance from \( A \) to the line \( BC \).

4. You are given segments of length \( a, b, c, \ldots \) and if needed a segment of length 1. Construct the following quantities and indicate if you need to use the unit segment.
   i) \( a\sqrt{2} \)
ii) $\sqrt{2a}$
iii) $\frac{a^2c}{br}$
iv) $\frac{1}{a} + \frac{1}{b}$
v) $\sqrt{a^2 + bc}$

5. Let $T(\vec{x}) = A\vec{x} + \vec{b}$ be an affine transformation.
   
   i) Give an example of affine transformation such that
   
   $$T\left(\begin{array}{c} 2 \\ 3 \end{array}\right) = \left(\begin{array}{c} -1 \\ 2 \end{array}\right)$$
   
   ii) List all affine transformations that preserve the origin and the $y$-axis.

   iii) Prove that an affine transformation that preserves both the $x$-axis and $y$-axis, preserves also the origin. List all such transformations.

   iv) Find an affine transformation $T$ that transforms the triangle with vertices $A = \left(\begin{array}{c} 2 \\ 3 \end{array}\right)$, $B = \left(\begin{array}{c} 4 \\ 3 \end{array}\right)$, $C = \left(\begin{array}{c} 4 \\ 6 \end{array}\right)$ into the standard triangle (vertices $\left(\begin{array}{c} 0 \\ 0 \end{array}\right)$, $\left(\begin{array}{c} 1 \\ 0 \end{array}\right)$, $\left(\begin{array}{c} 0 \\ 1 \end{array}\right)$).

6. Prove using affine geometry that the medians in a triangle meet in a single point.