In the following exercises $X$ is a locally compact Hausdorff space.

**Problem 1.** If $\mu$ is a Radon measure and $f \in L^1(\mu)$, show that $\nu(E) = \int_E f \, d\mu$ is a Radon measure.

**Problem 2.** If $\mu$ is a Radon measure and $f \in L^1(\mu)$ is real-valued, show that for every $\epsilon > 0$ there are an l.s.c. function $g$ and a u.s.c. function $h$ such that $h \leq f \leq g$ and $\int (g - h) \, d\mu < \epsilon$.

**Problem 3.** If $\mu$ is a positive Radon measure on $X$ with $\mu(X) = \infty$, show that there exists $f \in C_0(X)$ such that $\int f \, d\mu = \infty$. Consequently, every positive linear functional on $C_0(X)$ is bounded.

**Problem 4.** If $\mu$ is a $\sigma$-finite Radon measure on $X$ and $\nu \in M(X)$, let $\nu = \nu_1 + \nu_2$ be the Lebesgue decomposition of $\nu$ with respect to $\mu$. Show that $\nu_1$ and $\nu_2$ are Radon.

**Problem 5.** Show that a sequence $\{f_n\}$ in $C_0(X)$ converges weakly to $f \in C_0(X)$ iff $\sup_n \|f_n\| < \infty$ and $f_n \to f$ pointwise.

**Problem 6.** Find examples of sequences $\{\mu_n\}$ in $M(\mathbb{R})$ such that:

1. $\mu_n \to 0$ vaguely, but $\|\mu_n\| \not\to 0$.
2. $\mu_n \to 0$ vaguely, but $\int f \, d\mu_n \not\to 0$ for some bounded measurable $f$ with compact support.
3. $\mu_n \geq 0$ and $\mu_n \to \mu$ vaguely, but, there exists $x \in \mathbb{R}$ such that $F_n(x) \not\to F(x)$.

**Problem 7.** Let $\mu$ be a Radon measure on $X$ such that every open set has positive measure. Show that for each $x \in X$ there is a sequence $\{f_n\}$ in $L^1(\mu)$ which converges vaguely in $M(X)$ to the point mass at $x$. 