MATH 308, SPRING 2021 PRACTICE FINAL EXAM

MAY 18

Each problem is worth 10 points.

Problem 1. Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 4 \end{pmatrix}$.

a. Determine the eigenvalues and eigenvectors of A.

b. Calculate e^{A^2}.

Problem 2. Given the matrix $A = \begin{pmatrix} 2 & 0 & 2 \\ 0 & 1 & 1 \\ 2 & 1 & 3 \end{pmatrix}$.

a. Determine the null space and rank of A.

b. Calculate the eigenvectors and eigenvalues

c. Give an expression for $(I - tA)^{-1}$ that is valid for t in a neighborhood of 0.

Problem 3. Solve the ODEs:

a. $y'' - 2y' + y = e^x$.

b. $y'' + 3y' + 2y = 1 + e^x$.

c. $y'' - 4y = e^{2x} + 5\cos x$.

Problem 4. Solve the ODEs:

a.

$$x' + 2y = 0$$
$$y' + x + y = 0$$

b.

$$x' + y' = y$$
$$x' + 2y' = x.$$

Problem 5. Discuss the stability of the equilibrium solutions to

$$x' = -x(x^2 + y^2 - 1), \qquad y' = -y(x^2 + y^2 + 1).$$

What can you say about $\frac{y}{x}$.

Problem 6. Solve $y'' + xy' + y = 0$ in power series.

Problem 7. Calculate the Fourier series of the two-periodic function f given by $f(x) = \frac{1}{2} - |x|$ on $[-1, 1]$. Thus, or otherwise, solve the wave equation $a^2 u_{xx} = u_{tt}$, $u\left(-\frac{1}{2}, t\right) = u\left(\frac{1}{2}, t\right) = 0$, $u(x, 0) = 0$, $u_t(x, 0) = f(x)$.