MATH 308, SPRING 2021 MIDTERM 2

MARCH 14

Each problem is worth 10 points.

Problem 1.

a. Find the general solution of $y''' - 3y'' + 3y' - y = 0$.

b. Find the general solution of $y''' - 3y'' + 3y' - y = e^{2t}$.

c. Suppose $mx'' + kx' + hx$ describes the motion of a particle with mass m, force h and friction coefficient k all positive. Find the general solution of motion and explain the concept of critical damping k.

Problem 2. Solve the following ODE by Laplace transform.

$$y'' + 2y' + 2y = \cos t, \qquad y(0) = 1, y'(0) = 1.$$

Problem 3. Show that if the entries in an $n \times n$ matrix $A(t) = (a_{ij}(t))$ are differentiable functions of a real variable t, then the derivative of $\det(A(t))$ is computed by differentiating the entries of one row of $A(t)$ at a time and adding the resulting n determinants.

Problem 4. Solve the following systems of ODEs.

a.

$$x'' - 3x - 2y'' = 0,$$
$$x'' - y'' + 2x = 0.$$

b.

$$x'' - x + y' + y = 0,$$
$$x' - x + y'' + y = 0.$$

Problem 5. Write the van der Pol equation $x'' + \alpha(x^2-1)x' + x = 0$ as

$$\begin{aligned} x' &= y, \\ y' &= -x - \alpha(x^2-1)y. \end{aligned}$$

Find the linearization near $(0,0)$ and discuss the behavior there.

Problem 6. Solve the initial value problem

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} e^{2t} \\ 2e^{2t} \end{pmatrix}, \qquad \begin{pmatrix} x(0) \\ y(0) \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \end{pmatrix}.$$