MATH 141, FALL 2016, HW5

DUE IN SECTION, OCTOBER 4

Problem 1. Express each of the following complex numbers in the form $a + bi$.

(1) $e^{\pi i/2}$
(2) $2e^{-\pi i/2}$
(3) $3e^{\pi i}$
(4) $-e^{-\pi i}$
(5) $i + e^{2\pi i}$
(6) $e^{\pi i/4}$
(7) $e^{\pi i/4} - e^{-\pi i/4}$
(8) $\frac{1-e^{\pi i/2}}{1+e^{\pi i/2}}$

Problem 2. For each of the following functions, sketch the graph of f in polar coordinates and compute the area of the radial set over the interval specified.

(1) *Spiral of Archimedes*: $f(\theta) = \theta$, $0 \le \theta \le 2\pi$
(2) *Circle tangent to y-axis*: $f(\theta) = 2\cos\theta$, $-\pi/2 \le \theta \le \pi/2$
(3) *Rose petal*: $f(\theta) = \sin 2\theta$, $0 \le \theta \le \pi/2$
(4) *Four-leaved rose*: $f(\theta) = |\sin 2\theta|$, $0 \le \theta \le 2\pi$
(5) *Lazy eight*: $f(\theta) = \sqrt{|\cos\theta|}$, $0 \le \theta \le 2\pi$
(6) *Cardioid*: $f(\theta) = 1 + \cos\theta$, $0 \le \theta \le 2\pi$.

Problem 3. Compute the integral

$$\int_0^9 \sqrt{1+\sqrt{x}}dx.$$

Problem 4. We have defined π to be the area of a unit circular disk. Calculate in terms of π

$$\int_{-2}^2 (x-3)\sqrt{4-x^2}dx.$$

Problem 5. Calculate the average values of $\sin x$ and $\sin^2 x$ over the interval $[0, \pi/2]$.

Problem 6. Consider a rod of length L placed on the non-negative x-axis with one end at the origin. For each of the following mass-distributions ρ of the rod, determine the center of mass, moment of inertia and radius of gyration.

(1) $\rho(x) = 1$ for $0 \leq x \leq L$
(2) $\rho(x) = 1$ for $0 \leq x \leq \frac{L}{2}$, $\rho(x) = 2$ for $\frac{L}{2} < x \leq L$
(3) $\rho(x) = x$ for $0 \leq x \leq L$
(4) $\rho(x) = x$ for $0 \leq x \leq \frac{L}{2}$, $\rho(x) = \frac{L}{2}$ for $\frac{L}{2} \leq x \leq L$.

Note: for these definitions, see Apostol pp. 118-119, or the slides from Lecture 6, which have been updated.

Problem 7. Let $f : [a, b] \to [a, b]$ be continuous. Show that there exists $x \in [a, b]$ solving $f(x) = x$.

Problem 8. A set A of real numbers is dense if every open interval contains a point of A. Show that if f is continuous and $f(x) = 0$ for all x in a dense set of points, then $f(x) = 0$ for all x.

Bonus Problem. Let $f(0) > 0$ and $f(1) < 0$. Furthermore, assume that there exists a continuous function g on $[0, 1]$ such that $f + g$ is non-decreasing. Prove that there exists $x \in (0, 1)$ with $f(x) = 0$.

Bonus Problem. Define $g(x)$ on $[0, 1]$ by $g(x) = 0$ if x is irrational, and for rational $x = \frac{p}{q}$, $q > 0$ in lowest terms, $g(x) = \frac{1}{q}$. Show that g is Riemann integrable and $\int_0^1 g(x)dx = 0$.